



DEFINITIONS · THEOREMS · PROOFS · PAPERS

How to Read Mathematics

Definitions, Theorems, Proofs, and Papers for
Students

GAURAV TIWARI

How to Read Mathematics

Definitions, Theorems, Proofs, and Papers
for Students

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Preface

You're on page 41.

You've read the same paragraph four times. Every word in it is a word you know. The sentence is grammatical. And nothing's happening. No picture forms, nothing clicks, and the clock says you've been at this desk for fifty minutes and covered a page and a half.

So you draw the obvious conclusion. You're not smart enough for this.

You're wrong, and I can tell you exactly where. Mathematical prose isn't written to be read the way you read everything else. It's compressed, on purpose. A definition is a folded thing that means nothing until you unfold it. A theorem hides half its content in the order of its quantifiers. A proof is the tidy final draft of a messy discovery, with the scratch work deleted, and that deleted scratch work is exactly what would have made it obvious.

Nobody tells you this. You get handed the compressed object, you get graded on whether you can decompress it, and the decompression procedure is left as an exercise for the reader.

This book is the procedure.

It's the companion to *How to Write Mathematics*, and the two halves fit the way you'd expect. That one is about compressing your thinking so a reader can follow it. This one is about unfolding somebody else's. You don't need to have read it first, and you don't need any mathematics past what your own courses are teaching you right now. Nearly every worked passage here runs on first-course material: even integers, $\sqrt{2}$, sequences, continuity. The few that don't, including a research abstract and a metric-space definition in Appendix A, are there to show the moves still work when the mathematics is above you. Most examples come from a first analysis course, and a few use first ideas about graphs and groups; skip any you haven't met yet. The reading lessons are what transfer, so I've kept the mathematics out of their way.

How to use it. Part I is why the page resists you and what a mathematical text is actually made of. Parts II, III, and IV take the three objects

you'll meet ten thousand times, the definition, the statement, and the proof, and hand you a procedure for each. Part V scales up to chapters, whole books, and research papers. Part VI is about the pen in your hand and the people in the room.

Read it straight through if you like. But it's a toolbox, not a novel. Stuck on a definition right now? Go to Chapter 4 and run the drill on the page in front of you. Proof refusing to land? Chapter 10 tells you which of the three readings you're attempting and which one you actually need. Stuck and cross about it? Chapter 13 is a decision tree that ends with you doing something useful instead of reading the paragraph a fifth time. The appendices are working tools. Keep them close, write on them.

Two warnings, both honest.

This book won't make mathematics fast. Nothing makes mathematics fast. What it does is make your slowness productive, which is a different thing and a better one, and it stops you burning forty minutes in a rereading loop that was never going to work.

And it's going to ask you to write while you read. Every technique in here needs a pen. Reading mathematics with your hands in your pockets isn't reading mathematics. It's watching mathematics, and watching doesn't transfer.

That panic on page 41 isn't a verdict on you. It's a signal that the page is folded up and nobody ever taught you how to open it.

Let's open it.

Gaurav Tiwari
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Part I

Why the Page Resists You

Once a good building is finished, its scaffolding should no longer be visible.

Carl Friedrich Gauss, as reported by Sartorius von Waltershausen

Gauss zum Gedächtniss, 1856; author's translation

1

The Page That Won't Let You In

Here's an experiment I run with students who tell me they're bad at mathematics. I hand them a page from a real analysis textbook, set a timer for fifteen minutes, and ask them to read it. Not solve anything. Just read.

Fifteen minutes later, almost every one of them has done the same thing.

They read the definition at the top and got roughly nothing. Read it again, got roughly nothing again. Moved down to the theorem, because the definition wasn't going anywhere. Recognized none of the notation. Moved to the proof, because it looked like it might explain things. Followed two lines, hit a step that said *and therefore*, stopped, went back to the definition, and started the loop over.

They're not lazy and they're not slow. They're reading mathematics the way they read everything else, and that's the one thing that can't work.

1.1 The compression problem

Ordinary prose has slack in it. A paragraph in a history book says its main idea two or three different ways, gives you an example, tells you what's coming next. You can skim it, miss a third of the words, and still walk out with the argument. That redundancy is a gift from the author, and you've been trained on it since you were five.

Mathematical prose has the redundancy stripped out. It's written to be *correct* and *complete*, not to be absorbed at speed. One printed line can carry six separate ideas, each of which the author expects you to unfold on your own time with your own pen.

When a textbook says

Since f is continuous on the compact set K , it attains its bounds.

that’s thirteen words, and nobody on earth reads it in thirteen words’ worth of effort. Figure 1.1 shows what’s actually sitting inside it.

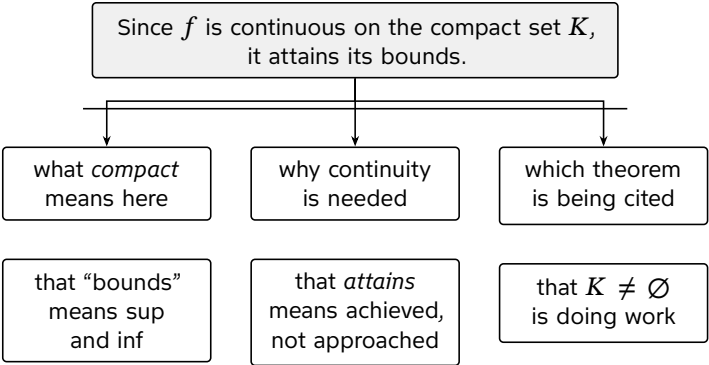


Figure 1.1: One printed line, unfolded. Everything below the rule is content the author expects you to supply.

Six unfoldings. And that’s a friendly sentence, written by somebody being kind to you.

It’s not badly written. It’s written for a reader who unfolds, and unfolding is a learned skill that nobody sat you down and taught.

Mathematical text is compressed on purpose. Reading it means decompressing it, on paper, with your own hand. The compression isn’t the author being unkind. It’s the price of being exact.

FROM THE FIELD

Thurston Writes Too Tersely and Empties a Field

In the early 1970s William Thurston was a young topologist working on foliations, the subject of his 1972 Berkeley thesis. He proved striking theorems faster than he could write them up, and the papers he did publish followed the usual dense house style. They leaned on back-ground that only a few insiders shared, and now and then they dropped a one-line insight with no explanation at all. Within a couple of years, people were advising graduate students to stay away from foliations

because Thurston was “cleaning it out.” The students listened, and the field emptied. Looking back in 1994, Thurston decided the subject had not been exhausted at all. His papers had simply been too hard to get into, and readers had wanted their own understanding of the ideas, not just a pile of proved results to cite. It shows the first reading skill in this book: when a page seems impossible, suspect compression, and go looking for the background the author assumed you already had.

Sources: W. P. Thurston, *On Proof and Progress in Mathematics*, 1994; MacTutor, *William Thurston*

1.2 Four ways students read, and why three of them fail

I’ve watched the same four reading styles show up over and over. Three are dead ends. Work out which one is your default, because you can’t stop doing something you can’t see yourself doing.

The passive read

Eyes move, pages turn, nothing gets written down. This feels like studying and produces nothing, because comprehension in mathematics gets tested by production, not recognition. You’ll finish a passive read of a chapter feeling fine about it and be unable to state a single definition from it an hour later.

The tell: you’ve been reading for thirty minutes and your pen hasn’t moved.

The linear read

You start at the first word and refuse to move on until you’ve understood it, so you never reach the paragraph three pages later where the author explains what they were doing. Mathematics texts aren’t written linearly. They’re written in layers, and the payoff for the definition on page 41 is often the example on page 44.

The tell: twenty minutes on the same object, and you haven’t looked ahead once.

The symbol-skip

Your eye slides over $\forall \epsilon > 0$ the way it slides over a long foreign name in a novel. It registers as *some symbols* and you move on. Since the whole meaning of most mathematical sentences lives in the symbols and their order, you've just skipped the sentence.

The tell: you can't read the line out loud, in English, without stumbling.

The active read

You have a pen. You're writing while you read: examples in the margin, the definition restated in your own words, a small picture, a question you can't answer yet. You look ahead when you're stuck, then come back. You say symbols out loud. You're slower per page and much faster per concept.

Everything in this book is a technique for making that fourth one automatic.

1.3 The rereading loop

The most expensive habit in student mathematics is the rereading loop, and it's expensive precisely because it feels like work.

You read a paragraph, don't get it, and read it again with a bit more determination. The second pass usually adds nothing, because the reason you missed it the first time wasn't insufficient attention. It was a missing piece somewhere else: a definition you never unfolded, a symbol you can't say out loud, an example you never built.

Determination can't supply a missing example. Only construction can.

Look at what the working exits have in common. Every one of them changes the page in front of you or the sound in the room. Not one of them is *concentrate harder*.

So when you catch yourself on that second pass with nothing written down, that's the moment. Stop reading. Start producing.

1.4 What reading mathematics actually is

Here's the definition that runs through the rest of this book.

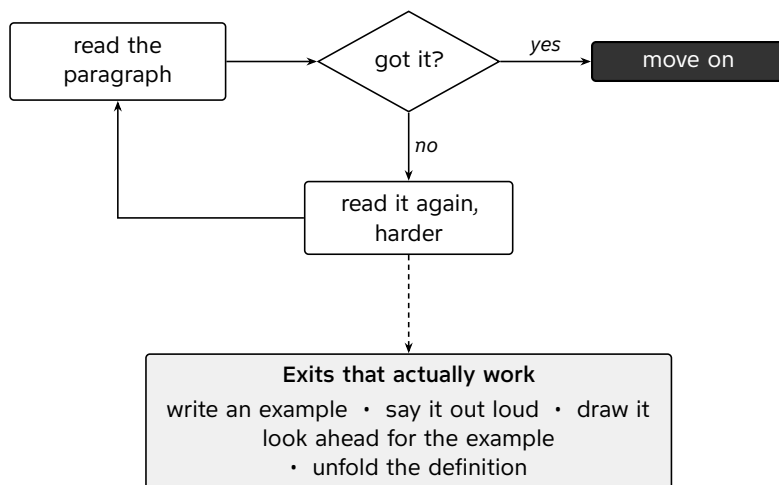


Figure 1.2: The rereading loop has no exit inside itself. Every way out is an action with a pen.

To read a piece of mathematics is to be able to reconstruct it: to state the definition in your own words, produce an example and a non-example, say why each hypothesis is there, and rebuild the proof's skeleton without looking.

High bar, and deliberately behavioral. Notice it never mentions understanding, insight, or feeling comfortable. Those are unmeasurable and they lie to you. Students feel comfortable with things they can't reproduce all the time (it's one of the most common self-deceptions in the subject, and it usually surfaces about six minutes into an exam).

Every technique in this book converts the vague question *do I understand this?* into something with a testable answer, like *can I write down a function that satisfies the first two hypotheses and fails the third?*

The bar's lower than it looks, by the way. Reconstruction is mostly bookkeeping once you know what to record, and the recording happens while you read, not in a panicked pass the night before.

TIP Count your rereads

Put a small tick in the margin every time you reread a sentence. Three ticks on one line is the signal to stop reading and start producing: say a symbol aloud, build an example, or unfold a definition.

1.5 What this book will and won't do

It won't teach you analysis, algebra, or topology. Your courses own that.

It won't make you fast. Five pages an hour is a respectable rate for a working read of a serious text, and it always will be.

And it won't save you from being stuck, because being stuck is the normal working state of everyone who does mathematics, including whoever wrote the book you're stuck on.

What it will do is give you a procedure for each of the three objects you'll meet ten thousand times. Definitions get unfolded (Part II). Statements get interrogated before you believe them. Proofs get read in a specific order, at one of three depths, on purpose instead of by accident. Then Part V scales all of it up to chapters, whole books read alone, and research papers, which are a different animal with their own rules.

Start where your pain is. Definition refusing to open? That's Chapter 4. Proof? Chapter 10. Just stuck and cross? Go straight to the protocol in Chapter 13 and come back here after.

Next, though, the map. Before you can read a mathematical text efficiently, you need to know what its pieces are and which of them hold the weight, because they don't all deserve the same attention and most students spend theirs in exactly the wrong places.

READ SOMETHING TODAY

Take the page you're stuck on right now. Ten minutes, pen in hand, and write down every symbol you can't say out loud in English. Don't try to understand anything. Just build the list. That list, not the page, is your actual work for today, and it's the start of your notation table.

B

The Unpacking Drill

The card on the next page is the whole drill from Chapter 4. Photocopy it, or copy it onto the inside cover of your notebook, and run it on every definition that matters. Here's what a full run looks like, on the definition of a bounded sequence: (a_n) is bounded if there exists $M > 0$ with $|a_n| \leq M$ for all n .

1. **Say it.** "Some single positive number is at least as big as the size of every term."
2. **Types.** (a_n) is a sequence of reals. M is a positive real, not a term and not indexed by n .
3. **Quantifiers.** $\exists M \forall n. M$ comes first, so one M has to work for the whole sequence.
4. **Smallest example.** $a_n = 1$, with $M = 1$.
5. **Non-example.** Negation: $\forall M \exists n$ with $|a_n| > M$. Read as a recipe, it builds $a_n = n$.
6. **Boundary.** $a_n = \sqrt{n}$: its steps shrink to zero and it's still unbounded.

Six lines, about eight minutes, and the definition is yours.

THE UNPACKING DRILL*8–15 minutes***1 Say it out loud** in English, no symbols.

If you stumble, the problem is notation, not the concept. Build the notation table first (Ch. 6; App. E has the common symbols).

2 List every object and its type.

Two columns: the symbol, and what kind of thing it is. A number, a set, a function, a sequence, an element of what?

3 Mark the quantifiers, in order.

Draw an arrow from each variable to every variable quantified before it. Those are the only things it may depend on. Read it as a game: they choose the $\forall$ s, you answer with the $\exists$ s.

4 Build the smallest example.

The dullest object that works. Check it clause by clause, with the definition in view.

5 Build a non-example.

Negate the definition, then read the negation as instructions. Name the clause that fails.

Negating: $\forall$ and $\exists$ swap, in the same order. Restrictions like $\varepsilon > 0$ and $n > N$ stay as written. Only the final statement gets negated. $\neg(P \Rightarrow Q)$ is $P \wedge \neg Q$. $\neg(P \wedge Q)$ is $\neg P \vee \neg Q$. $\neg(P \vee Q)$ is $\neg P \wedge \neg Q$.

6 Push to the boundary.

What's the closest thing that *just* fails? That object is the definition.

Done when you can say the definition without symbols, and you hold one example, one non-example and one near miss, each checked against the definition.

If step 4 defeats you, one of three things is true, and none of them is that you aren't clever enough:

- 1. An earlier definition is still folded. Go back one level and run the drill there. Recurse until you can build something.
- 2. The examples are genuinely hard, and the author knows it. Read ahead; the example is often in the next paragraph.
- 3. You're missing notation. Build the notation table.

The two failure modes and their fixes:

Symptom	Name	Fix
"I thought N bounded a_n "	type confusion	step 2, 90 seconds
"I read $gHg^{-1} = H$ as $ghg^{-1} = h$ "	type confusion	step 2: a set, not an element
"I let N depend on n "	scope confusion	step 3, 2 minutes

Keep Reading

That was Chapter 1. The full book has 21 more chapters and 8 appendices, and they build the procedure one kind of text at a time. The drill card you just read is one piece of it.

- **Definitions:** the 6-step unpacking drill, the example bank and notation you can say out loud.
- **Statements:** reading a theorem before its proof and testing it against the edges.
- **Proofs:** 3 ways to read a proof, the skeleton, the reconstruction test, a stuck protocol and a single theorem read start to finish.
- **Longer texts:** chapters, whole books on your own, research papers and reading above your level.
- **With a pen, and with people:** what to write while you read and how to use lectures, study groups and office hours.
- **The toolkit:** 7 reading makeovers, the drill and the stuck protocol as cards you can photocopy, time budgets for each move, tips at a glance and answers to the questions in the text.

The full book is \$5.99. You get 2 DRM-free PDFs of the same book, 6×9 and A4, ready to download right after checkout.

gauravtiwari.org/product/how-to-read-mathematics

The drill card is yours to keep. You can print it, photocopy it and run it on the next definition that stops you.

You're not slow. The page is folded.

You've read the same paragraph four times. Every word in it is a word you know, and nothing is happening. So you draw the obvious conclusion: you're not smart enough for this.

You're wrong, and this book can tell you exactly where. Mathematical prose is compressed on purpose. A definition is a folded object that means nothing until you unfold it, a theorem hides half its content in the order of its quantifiers, and a proof is the tidy final draft of a messy discovery with the scratch work deleted. The decompression is rarely taught.

This book is the procedure. A repeatable set of moves for the three objects you'll meet ten thousand times, and for the chapters, books, and research papers they live in.

For students in a first proof course and everyone within shouting distance of one, and for anyone working through a serious text alone.

- ✓ The six-step drill that opens any definition, on a photocopiable card
- ✓ Three ways to read a proof, and how to tell which one you're actually doing
- ✓ The drop-one test: what each hypothesis is quietly holding back
- ✓ A stuck protocol that always ends in an action, never in rereading
- ✓ Honest time budgets, so slowness stops reading as failure
- ✓ Seven reading makeovers, and one whole theorem read from start to finish

Gaurav Tiwari is an educator and developer with an M.Sc. in Mathematics. He writes for students at gauravtiwari.org and is the author of *How to Write Mathematics and LaTeX for Students*.