ARITHMETIC

Whole Numbers

The whole numbers and their properties, place value, order of operations, and a 10-question practice set with full solutions.

SUBJECT	LEVEL	INCLUDES
Mathematics	High school and early college	Practice set, answer key, revision sheet

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INSIDE

Whole numbers are one of the first building blocks you encounter in mathematics, yet they show up everywhere, from counting everyday objects to performing complex calculations. In this guide, you will start with natural numbers, understand what makes whole numbers different, and then work through place value, rounding, comparing, and all four arithmetic operations (addition, subtraction, multiplication, and division) with step-by-step examples and practice exercises.

Also, see: Dedekind's Theory of Real Numbers

Natural numbers

Natural numbers are regular numbers that we use to count objects or indicate the serial number of an object. For example, you can count the number of apples in a bag, fingers on your hand, or trees in an orchard. We can write any natural number with the help of the ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Examples of natural numbers are 5, 12, 76, 145, etc.

The smallest natural number is 1, and there is no biggest natural number because we can add 1 to any natural number and get a bigger number as a result. For example, you may think that 50 is the biggest natural number, but by adding 1 to it, you will obtain a bigger number, 51.

What is a whole number?

Whole numbers are natural numbers along with zero (0).

Thus, all natural numbers such as 1, 2, 3, etc. are whole numbers as well, whereas 0 (zero) is a whole number but not a natural number.

Zero is not a natural number because we cannot use it for counting. Zero is, in a way, a mathematical representation of nothing. For example, if someone has no money, we can say that they actually have zero money.

KEY TAKEAWAY

Every whole number is either zero or a natural number. The set of whole numbers is written as W = {0, 1, 2, 3, 4...}. You can think of whole numbers as the complete collection of non-negative integers, which makes them foundational to all number systems in mathematics.

What is the place value of a whole number?

In a whole number, the value of a digit depends on its place (position) in the number. The rightmost digit is known as "ones", next to the left is "tens", followed by hundreds, thousands, ten thousands, hundred thousands, millions, ten millions, and so on.

Example 1, Determine the place value of each digit in 5307.

Answer:

5, thousands

3, hundreds

0, tens

7, ones

Example 2, Determine place value of each digit in 23846921523.

Answer:

2, ten billions

3, billions

8, hundred millions

4, ten millions

6, millions

9, hundred thousands

2, ten thousands

1, thousands

5, hundreds

2, tens

3, ones

In this example, we've worked with a pretty long number. It is better to write such numbers in standard notation by separating the digits into groups of three, starting from the right: 23,846,921,523.

Example 3 - Write 56987 in standard notation.

Answer: 56,987.

Finally, notice that some numbers have no thousands. For example: 417. However, you can easily add it to them. In this case, 417 is the same as 0417. Therefore, the thousand here is 0.

You can add as many leading zeros (and ignore them) as you want to the whole number. For example, 856 is same as 0856 or 000000856.

However, remember that we can ignore only leading zeros. 709 is *not*the same as 79, although 00709 is same as 709.

Now take up a pen and paper and solve the problems given below. If you want to strengthen your foundation further, check out the best algebra books for structured practice.

Exercise 1 - Determine place value of each digit in 12574.

Answer:

1, ten thousands

2, thousands

5, hundreds

7, tens

4, ones

Exercise 2 - Determine the place value of each digit in 42586921.

Answer:

4, ten millions

2, millions

5, hundred thousands

8, ten thousands

6, thousands

9, hundreds

2, tens

1, ones

Exercise 3 - Write 7487493 in standard notation.

Answer: 7,487,493.

Rounding whole numbers

Sometimes we have to round whole numbers to the tens, hundreds etc. For that, we must know the place value of the whole number in question. Let's look at some examples to learn how to do this.

Example 1 - Round 147 to the nearest ten.

First, you need to find the tens in 147: it is 4.

After that, consider the digit immediately to the right of 4: it is 7. Since 7 is greater than or equal to 5, we must increase 4 by 1 (resulting in 5) and change all digits to the right of 4 to zeros.

So, 147 rounded to ten is 150.

Answer: 150.

Example 2, Round 546 to the nearest ten.

First, find the tens in 524: it is 2.

Now consider the digit immediately to the right of 2: it is 4.

Since 4 is less than 5, we will leave 2 and change all the digits to the right of 2 to zeros.

So, 524 rounded to ten is 520.

Answer: 520.

Let's now see how to round a whole number to the nearest hundred.

Example 3, Round 847 to the nearest hundred.

First, find the hundreds in 847: it is 8.

Now consider the digit immediately to the right of 8: it is 4.

Since 4 is less than 5, we need to leave 8 and change all the digits to the right of 8 to zeros.

So, 847 rounded to hundred is 800.

Answer: 800.

Example 4, Round 8371 to the nearest hundred.

First find the hundreds in 8371: it is 3.

After that, consider the digit immediately to the right of 3: it is 7.

Since 7 is greater than or equal to 5, we will increase 3 by 1 (resulting in 4) and change all the digits to the right of 3 to zeros.

So, 8371 rounded to hundred is 8400.

Answer: 8400.

Now, let's see how to round a whole number to the nearest thousand.

Example 5, Round 952 to the nearest thousand.

First, find the thousands in 952. Although there is no apparent thousand here, we can easily add it as discussed before. We can write 952 as 0952 (recall that we can ignore the leading zeros). Therefore, the thousand here is 0.

Now consider the digit immediately to the right of 0: it is 9. Since 9 is greater than or equal to 5 then we increase 0 by 1 (resulting in 1) and change all digits to the right of 1 to zeros.

So, 952 rounded to thousand is 1000.

Answer: 1000.

The final example is trickier because we need to consider other digits in the number as well.

Example 6, Round 3289791 to the nearest thousand.

Start by finding the thousands in 3289791: it is 9.

Next, consider the digit immediately to the right of 9: it is 7.

Since 7 is greater than or equal to 5, we must increase 9 by 1 (resulting in 10) and change all digits to the right of 9 to zeros.

By doing this, we have obtained 10 which is two-digit number. However, we can only replace 9 with one digit.

We can solve this issue by leaving 0 and adding the remaining 1 to the digit immediately to the left: $8+1=9$.

Thus, 3299791 rounded to thousand is 3290000.

Answer: 3290000.

Similarly, you can round a whole number to any place. Find the digit on the required place and examine the digit immediately to its right. If it is greater than or equal to 5, increase the required digit by 1 and change all the digits to the right of this digit to zeros. Otherwise, leave the digit as it is and change all the digits to its right to zeros.

> TIP
>
> A quick way to remember the rounding rule: look one place to the right. If that digit is 5 or more, round up. If it's 4 or less, round down. This same rule works whether you're rounding to tens, hundreds, thousands, or any other place value of a whole number.

Now try to consolidate your knowledge by solving the problems below.

Exercise 1, Round 138 to the nearest ten.

Answer: 140.

Exercise 2, Round 53 to the nearest ten.

Answer: 50.

Exercise 3, Round 12751 to the nearest hundred.

Answer: 12800.

Exercise 4, Round 124,516 to the nearest thousand.

Answer: 125,000.

Exercise 5, Round 1961274 to the nearest million.

Answer: 2,000,000.

Exercise 6, Round 2491 to the nearest ten.

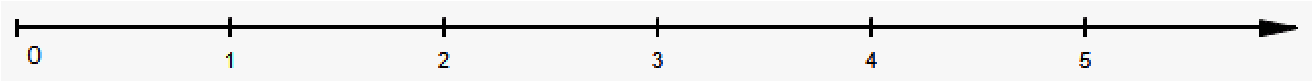
Answer: 2500.

Exercise 7, Round 230,982 to the nearest ten.

Answer: 231,000.

Whole numbers on a number line

The number line for whole numbers is a horizontal straight line where each whole number is depicted as a specially-marked point, evenly spaced on the line.



Although this image shows only six whole numbers (from 0 to 5), the arrow indicates that the number line actually includes all whole numbers.

The number line is drawn in such a way that the larger whole number always lies to the right of the smaller one.

For example, since 4 is bigger than 3, it is drawn to the right of 3.

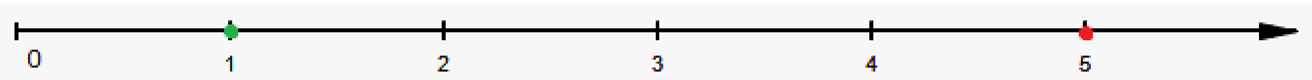
Comparing whole numbers

We can compare whole numbers as shown below:

COMPARISON	NOTATION
Greater than	$>$
Less than	$<$
Equal to	$=$
Not equal to	$\neq$
Greater than or equal to	$\geq$
Less than or equal to	$\leq$

The first way of correctly comparing whole numbers is using the number line. As we discussed before, the greater number lies to the right of a smaller number on the number line.

For example, since 5 lies to the right of 1, 5 is greater than 1, i.e., $5>1$ or $5\geq1$.



In this case, we can also say that 1 is less than 5 or $1<5$. This implies that we can switch numbers and inequality.

Let us now learn how to determine which sign to apply while comparing whole numbers.

Example 1, Choose the correct sign(s) for comparing 3?4.

Since 3 is less than 4 and is not equal to 4, the correct signs are $3<4$, $3\leq4$, and $3\neq4$.

Example 2, Choose the correct sign(s) for comparing 5?5.

In this case, we obviously need to use equality signs. Thus, we can say that $5=5$, $5\geq5$, or $5\leq5$.

It is easy to compare small numbers this way. However, we obviously cannot draw a number line for huge numbers such as 5,356,985.

Fortunately, there is an easy way to determine which number is greater in such cases.

If the numbers have a different number of digits, add required number of leading zeros until they have the same number of digits.

Start by comparing the digits from left to right. If a digit in the first number is less than the digit on the same place in the second number, then the first number is less than second one. If the digits are equal, then proceed to examine the digit to the right of the earlier one.

Example 3, Compare 4567 and 12345.

In this case, the second number has five digits while the first has only four. Thus, we need to add a leading zero to the first number to make it 5-digit.

First number, 04567.

Second number, 12345.

Now let's compare the leftmost digits of both numbers: 0 and 1.

Since $0<1$, we can conclude that $4567<12345$.

Example 4, Compare 345,612 and 2,457.

Here, the first number has six digits while the second one has four digits. Thus, we need to add two leading zeros to second number: 2457 becomes 002457.

First number: 345612.

Second number: 002457.

You can see that the leftmost digit of the first number is 3, and that of the second number is 0.

Since $3>0$, we can conclude that $345{,}612>2{,}457$.

Example 5, Compare 8135 and 8136.

Since these numbers have the same number of digits, we don't have to add leading zeros to them.

First number: 8135.

Second number: 8136.

On comparing the leftmost digits (8 and 8), you can see that they are equal.

Now, move to the right and compare 1 and 1. Again, they are equal.

Move further to the right and compare 3 and 3; they are equal.

Move again to the right and compare 5 and 6. Since $5<6$, $8135<8136$.

Let us now practice a few more problems.

Exercise 1, Choose correct sign(s) to compare: 15?0.

Answer: $15>0$, $15\geq 0$, $15\neq 0$

Exercise 2, Choose correct sign(s) to compare: 10?19.

Answer: $10<19$, $10\leq 19$, $10\neq 19$.

Exercise 3, Compare 36589 and 167.

Answer: $36589>167$

Since the number of digits is not equal here, we must add leading zeros to the second number: 167 becomes 00167.

Exercise 4, Compare 4125 and 4200.

Answer: Since $1<2$, $4125<4200$.

Exercise 5, Compare 59734 and 59734.

Answer: These numbers are equal, i.e. $59734=59734$.

Adding whole numbers

Let us now understand how to add whole numbers.

Let's say you have a box containing 3 mangoes. If someone gave you another mango, how many mangoes do you have? Of course, you must know that the answer is 4. Therefore, we can see that $3+1=4$.

Thus, we can construe that addition is a process during which we gain something.

Let's consider another example: suppose you have 3*inyourpocket*.*Yourunclegivesyou*5 as a gift. So how much money do you have now? The answer is $8, so $3+5=8$.

Each number being added is known as the **addend**, and the result is called the **sum**.

So, in 3+5=8 both 3 and 5 are addends and 8 is the sum.

It is easy enough to add one-digit numbers, but how can we add numbers like 48, 379, or 2100? Don't worry; there is a reliable technique for that. Let us begin with a simple example.

Example 1, Calculate 56+23.

To solve this problem, let's write the numbers one under another:

56

23

Starting from right, add 6 and 3: the result is 9. Write it under 6 and 3.

56

23

9

Now go back to add 5 and 2: the result is 7. Write it under 5 and 2.

56

23

79

Therefore, 56+23=79.

The same applies for all numbers in general, regardless of the number of digits in them.

Example 2, Calculate 213+124.

Once again, let's begin by writing the numbers one under another:

513

124

Starting from the right, add 3 and 4: the result is 7. Write it under 3 and 4.

513

124

7

After that, add 1 and 2: the result is 3. Write it under 1 and 2.

513

124

37

Finally, add 5 and 1: the result is 6. Write it under 5 and 1.

513

124

637

Thus, 513+124=637.

Let us try out a tougher problem now.

Example 3, Calculate 948+197.

Again, we must write the numbers one under another.

948

197

First, let's add 8 and 7: the result is 15. Naturally, we need a 1-digit number here. Therefore, we write down 5 and remember 1:

948

197

5

Now, add 4 and 9: the result is 13. Remember 1 from before: therefore, the result is 14. Again, we need a 1-digit number, so take 4 and remember 1:

948

197

45

Finally, add 9 and 1 and remember 1 from before: the result is 11. Since we are on final step, we can just write 11.

948

197

1145

Thus, 948+197=1145.

So far, we have worked with numbers that have the same numbers of digits. If you have to add two numbers that have different numbers of digits, take number that has the lesser number of digits and add zeros in front of it until both of them have the same number of digits.

For example, let's say suppose you need to add 51 and 8674. In this case, we take the 2-digit number 51 and add zeros in front of it: 251 becomes 0051. Now we can easily add the two numbers.

The next example will make this technique clearer.

Example 4, Calculate 32+128.

Here, we first need to add a zero in front of 32: it becomes 032.

Now, we can add 032 and 128 using the standard addition technique.

So, 32+128=160.

Finally, let's see how to add more than two numbers.

Example 5, Find 349+23+568.

Our approach here is the same as adding two numbers.

First, we write 0 in front of 23: it becomes 023.

Now add 9 and 3: the result is 12. Write down 2 and remember 1.

Continue adding: 2+8 is 10, so we write down 0 and remember 1. However, since we already have another 1 from before, we now have 1+1=2 reserved 1s.

After that, start working on the second column. Take the borrowed 2 and add it to 4: the result is 6.

Next, add 2 to 6: the result is 8. Further add 6 to 8: the result is 14. Write down 4 and remember 1.

It's time to focus on the leftmost column. Take the borrowed 1 and add it to 3: the result is 4.

Now add 0 to 4; it remains unchanged. Finally, add 5 to 4: the result is 9.

Thus, 349+23+568=940.

Now that we've gone through plenty of examples, it's time for you to take up a pen and paper and solve the problems below.

Exercise 1, Find 21+42.

Answer: 63.

Exercise 2, Find 62+79.

Answer: 141.

Exercise 3, Find 418+73.

Answer: 491.

Exercise 4, Find 502+436+198.

Answer: 1136.

Exercise 5, Find 9123+45+78+698+1827.

Answer: 11771.

Subtracting whole numbers

Subtraction is, in a way, the inverse of addition. Let us now learn how to subtract whole numbers.

Let's say you have $10 in your pocket right now. Your friend has 30*withhim*, *anddecidestogiveyou*20. You obtain 20, *andnowyouhave*10+20 =30. However, you friend "loses" 20*andnowhas*30-20 =10.

Thus, we can think of subtraction as a process during which one loses something.

Consider another example: suppose you have 8 bananas. You eat three of them. How many bananas do you have left? The answer is 5, so 8-3=5.

In general, if we want to find the difference between two numbers a and b (by subtracting one from the other) c=b–a, then we actually need to find such a number c, so that b=a+c.

Also, for c=b–a, b is called the minuend, a is called the subtrahend, and c is known as the difference.

Thus, in 7-3=4, 7 is minuend, 3 is subtrahend and 4 is difference.

The technique for subtracting whole numbers is similar to the one for their addition.

Let us begin with a simple example.

Example 1, Calculate 24-13.

First, let us write the numbers one under another:

24

13

Starting from the right, subtract 3 from 4: the result is 1. Write it down under 4 and 3.

24

13

1

Now proceed to subtract 1 from 2: the result is 1. Write 1 under 2 and 1.

24

13

11

So, 24-13=11.

The same applies to all numbers in general, regardless of the number of digits in them.

Example 2, Calculate 542-120.

Again, first write down the numbers one under another:

542

120

Starting from the right, subtract 0 from 2: the result is 2. Write it down under 2 and 0.

Now subtract 2 from 4: the result is 2. Write it down under 2 and 0.

Finally, write 5-1=4 under 5 and 1.

Thus, 542-120=422.

Let's now check out a harder example now.

Example 3, Calculate 946-197.

Begin by writing the numbers one under another:

946

197

Now, subtract 6 from 7. But wait: it looks like we are subtracting the bigger number from the smaller one! To avoid this, add 10 to 6 (result is 16) and remember that you are borrowing 1. Now subtract 7 from 16: the result is 9.

Proceed to subtract 9 from 4. Again, we are trying to subtract the bigger number from the smaller one. Thus, add 10 to 4 (the result is 14) and subtract 9 from 14: the result is 5. Also, don't forget to subtract the borrowed 1: the result is 4.

Finally, subtract 1 from 9 and also subtract the borrowed 1: the result is 7.

Thus, 946-197=749. You can verify the result by checking that 946=749+197.

The next example demonstrates how to subtract numbers that have different numbers of digits.

If two numbers have different numbers of digits, then first take the number that has the smaller number of digits and add zeros in front of it until both of them have an equal number of digits.

For example, let's say you need to subtract 46 from 7942. In this case, we must take the 2-digit number 46 and place zeros in front of it: 46 becomes 0046. Now we can easily subtract 0046 from 7942.

Example 4, Calculate 276-31.

Begin by adding a zero in front of 31: it becomes 031.

Now, we can subtract 031 from 276 using the standard technique.

Thus, 276-56=289.

Now, let's see one last example of subtracting more than two numbers.

Example 5, Find 501-47-368.

This is essentially the same as subtracting just two numbers.

First, write 0 in front of 47: it becomes 047.

Now proceed to subtract 7 from 1. Obviously, you need to add 10 and remember that you have borrowed 1: 1+10-7=4.

Continue subtracting and subtract 8 from 4. Again, you must add 10 and borrow 1. But since you have already borrowed 1 previously, you have borrowed a total of 2. Now, calculate: 4+10-8=6.

It's time to work with second column: subtract 4 from 0. Again, we need to add 10 and borrow 1: 0+10-4=6.

Keep subtracting in the second column and subtract 6 from 6: the result is 0. Now subtract the borrowed 2, again, we need to borrow 1. Since we have two borrowed 1s at this point: 0+10-2=8.

Now, start working on the leftmost column. Subtract 3 from 5: the result is 2. Finally, subtract the borrowed 2: 2-2=0.

We thus obtain the final result: 086, which is just 86.

So, 501-47-368=86.

If you find this technique too difficult, you can opt for an easier two-step method. First, subtract 47 from 501 to get 454 and then subtract 368 from 454 to get 86.

Now try to solve the following problems by yourself:

Exercise 1, Find 58-31.

Answer: 47.

Exercise 2, Find 86-62.

Answer: 24.

Exercise 3, Find 473-51.

Answer: 422.

Exercise 4, Find 950-471-113.

Answer: 366.

Exercise 5, Find 7931-67-82-950-2318.

Answer: 4514.

Multiplying whole numbers

In this section, we will learn how to multiply whole numbers.

Let's say four of your friends give you 6 candies each. So what is the total number of candies you have? Obviously, you can easily calculate it using addition: $6+6+6+6=24$.

However, making such calculations is clearly a long and cumbersome procedure. Also, if you have, say, 100 friends who give you 6 candies each, you will have to add 6 a hundred times!

Clearly, the addition method is not practical here. Therefore, we use multiplication for such cases.

The multiplication of numbers a and b is: $a \cdot b=a+a+a+a+\ldots+a$ ("b" number of times.) For example: $2 \cdot 5=2+2+2+2+2=10$

$3 \cdot 8=3+3+3+3+3+3+3+3=24$

Thus, you can see that multiplication essentially tells us how many times to use a number in addition.

Each number being multiplied is known as a factor and the result is called the product.

Thus, in $2 \cdot 5=10$, both 2 and 5 are factors and 10 is the product.

Another notation for multiplication is: $\times$, so $a \cdot b$ and $a \times b$ are equivalent.

Conveniently, order does not matter in multiplication. For example, $2\times3=2+2+2=6$ and $3\times2=3+3=6$. You can clearly see that the result is the same in both cases.

For any two numbers a and b, we can say that $a\times b=b\times a$.

Some other interesting facts:

Zero multiplied by any number is zero. For example, $31\times0=0$, $0\times964=0$.

Any number multiplied by 1 is the number itself. For example, $12\times1=12$, $53\times1=53$, $1\times7548=7548$.

For the results of multiplication of some common numbers, you can refer to the multiplication table in the next section.

Let us now learn how to multiply whole numbers.

Example 1, Find 25×13.

First, write the numbers one under another.

25

13

Start with the digit at the bottom right and multiply it with all the digits of the first number. After that, multiply 1 (the digit at the lower left) with all digits of the first number, but start writing the result from the position of 1.

 25

 13

 75

25

You must now add these two resulting numbers. However, since the lower number is shifted, just imagine zeros on the places where digits are missing. After that, use the addition technique on both of them.

 25

 13

 75

250

325

So, $25\times13=325$.

Let's try out a harder problem.

Example 2, Multiply 852 by 697.

Again, write the two numbers one under another:

852

697

Multiply 7 with all digits of first number. When we multiply 7 and 2, the result is 14. Since we need only one digit, we take 4 and remember 1.

Next, multiply 7 and 5: the result is 35. Add the remembered 1 to get 36. Again, we have two digits, so take 6 and remember 3.

Now, multiply 7 and 8: the result is 56. Add the remembered 3 to get 59. We have two digits again, but since we are done with the last digit of the second number (i.e. 7), we can just write 59.

852

697

5964

Next, start working with 9. Multiply 9 by 2: the result is 18. Take 8 and remember 1.

Multiply 9 by 5: the result is 45. Add the remembered 1 to get 46. Take 6 and remember 4.

Multiply 9 by 8: the result is 72. Add the remembered 4 to get 76. We are already done with 9, so just write 76.

852

697

5964

7668

Finally, work with 6. Multiply 6 and 2: the result is 12. Take 2 and remember 1.

Multiply 6 by 5: the result is 30. Add the remembered 1 to get 31. Take 1 and remember 3.

Finally, multiply 6 by 8: the result is 48. Add the remembered 3 to get 51. We are done with 6, so simply write 51.

852

697

5964

7668

5112

Again, since the numbers are shifted, imagine zeros where the missing digits should be and add the numbers.

852

697

5964

76680

511200

593844

So, 852×697=593844

Now, the question is: how do we multiply numbers that have different numbers of digits? The procedure remains the same: just write the number with lesser digits under the number with more digits.

Let us learn this with the help of another example.

Example 3, Multiply 17 by 416.

Since 17 has the lesser number of digits, start by writing 17 under 416. As usual, multiply both of its digits with 416 and add the two resulting numbers.

We will do this example faster:

So, 17×416=7072.

Finally, let's see how to multiply more than two numbers. Although it requires a bit more work, it is quite easy as well.

Let's say you need to multiply 12, 47 and 65. Take 12 and 47. Multiply them: the result is 564. Now multiply 564 and 65: the result is 36660.

So, 12×47×65=36660.

Keep in mind that the order of numbers is not important here. You can easily take 47 and 65, multiply them, and then multiply result by 12 to get the same result.

Now take up your pen and paper and try to solve the problems below.

Exercise 1, Find 14×48.

Answer: 672.

Exercise 2, Find 69×31.

Answer: 2139.

Exercise 3, Find 405×76.

Answer: 30780.

Exercise 4, Find 612×134.

Answer: 82008.

Exercise 5, Find 571×42×87×3128.

Answer: 6526365552.

Multiplication table

Below is the 15 times table.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
3	0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45
4	0	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60
5	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75
6	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90
7	0	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105
8	0	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120
9	0	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135
10	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
11	0	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165
12	0	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
13	0	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195
14	0	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210
15	0	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225

We already know that we don't have to multiply numbers in any particular order (for example, $6\times2=2\times6=12$). Therefore, we actually don't need nearly a half of the above table, which makes it easier to remember.

Let's see how to read the multiplication table. Suppose you want to multiply 14 by 6. See how to find a value below.

	0	1	2	3	4	5	**6**	7	8	9	10	11	12	13	14	15
0	0	0	0	0	0	0	**0**	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	**6**	7	8	9	10	11	12	13	14	15
2	0	2	4	6	8	10	**12**	14	16	18	20	22	24	26	28	30
3	0	3	6	9	12	15	**18**	21	24	27	30	33	36	39	42	45
4	0	4	8	12	16	20	**24**	28	32	36	40	44	48	52	56	60
5	0	5	10	15	20	25	**30**	35	40	45	50	55	60	65	70	75
6	0	6	12	18	24	30	**36**	42	48	54	60	66	72	78	84	90
7	0	7	14	21	28	35	**42**	49	56	63	70	77	84	91	98	105
8	0	8	16	24	32	40	**48**	56	64	72	80	88	96	104	112	120
9	0	9	18	27	36	45	**54**	63	72	81	90	99	108	117	126	135
10	0	10	20	30	40	50	**60**	70	80	90	100	110	120	130	140	150
11	0	11	22	33	44	55	**66**	77	88	99	110	121	132	143	154	165
12	0	12	24	36	48	60	**72**	84	96	108	120	132	144	156	168	180
13	0	13	26	39	52	65	**78**	91	104	117	130	143	156	169	182	195
14	**0**	**14**	**28**	**42**	**56**	**70**	**84**	98	112	126	140	154	168	182	196	210
15	0	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225

Thus, $14\times6=84$. You can also verify that $6\times14=14\times6=84$.

Dividing whole numbers (long division)

Dividing whole numbers is, in a way, the inverse of multiplying whole numbers.

The result of dividing a number a by another number b is the number c=ab, such that $a=b\times c$. You'll encounter this relationship frequently when studying functions and their rules later on.

For example, since $12=3\times4$, then 4=123.

Division is denoted by the symbol a/b (shown as ab). Although we generally use this symbol, you can also use common symbols such as ÷ and :

Thus, 124, 12÷4, and 12:4 are equivalent.

To understand division, we must think of division as a process where we try to find out how many times a number (divisor) is contained in another number (dividend). The result of division is known as the quotient.

For example, since 20=5+5+5+5 or 20=5×4, we may conclude that the number 5 is contained four times in 20. Thus 205=4. Here, 5 is the divisor, 20 is the dividend, and 4 is the quotient.

Also, we must remember another important fact, it is not possible to divide a number by zero: a0 is undefined. We simply can't determine how many times zero is contained in any number.

For any number a, we can say that 0a =0 (the number a is contained 0 times in 0). For example, 010 =0; the number 10 is contained 0 times in 0.

It is quite easy to determine how many times a small number is contained in another number, but working with big numbers can be trickier. For example, can we determine how many times is the number 12 contained in 2184?

In this section, you will understand how to divide a whole number by another number. Let's start with a simple example.

Example 1, Find 86÷2.

First, write the numbers in this special form:

2)

First, we must divide 8 by 2. For that, think: how many 2s are there in 8? The answer is, there are four 2s in 8: 2+2+2+2=8., Therefore, 82 =4. Write down 4 and multiply it by 2: the result is 8. Write it down.

2)(4

-8

Now subtract 8 from 8: the result is 0.

2)(4

-8

0

Drag down 6 as shown below.

2)(4

-8_

06

Let's sum up what what we have done here: we've first done division, then multiplication, and then subtraction. Let's proceed further in the same way.

Divide 6 by 2: the result is 3. Now, multiply 2 and 3 to get 6.

2)(43

-8_

06

-6

Finally, subtract 6 from 6: the result is 0.

2)(43

-8_

06

-6

0

Since we have obtained zero and there are no more numbers to divide, we are done with the division.

So, $86 \div 2 = 43$.

Let's check out another example.

Example 2, Find $72 \div 3$.

Again, write the two numbers in special form:

3)

Now, check: how many 3s are in 7? $3+3+1=7$. So, there are two 3s and something extra in 7.

Write down 2. Now, multiply 3 by 2: the result is 6. Write it down.

3)(2

-6

Now, subtract 6 from 7: the result is 1.

3)(2

-6

1

Drag 2 down.

3)(2

-6_

12

Continue with 12; check how many 3s there are in 12. $3+3+3+3=12$: thus, there are four 3s in 12.

Now, multiply 4 and 3: the result is 12.

3)(24

-6_

12

-12

Subtract 12 from 12. Result is 0.

3)(24

-6_

12

-12

0

Since we obtained zero and there are no numbers left to divide, we are done with the problem.

So, $72 \div 3 = 24$

Finally, let's look at a harder example.

Example 3, Find $2184 \div 12$.

12)

Think: how many 12s are there in 2? The answer is zero; after all, 12 is greater than 2.

Therefore, we need to consider the next digit as well: take 21 instead of 2. How many 12's are there in 21? The answer is one: $12+9=21$. Write down 1 and multiply it by 12: the result is 12. Write it down.

12)(1

-12

Subtract 12 from 21: the result is 9.

12)(1

-12

9

Drag down 8.

12)(1

-12

98

Continue with 98; how many 12s are there in 98? The answer is 8: 12+12+12+12+12+12+12+12+2=98. Multiply 12 and 8: the result is 96.

12)(18

-12

98

-96

Subtract 98 from 96: the result is 2.

12)(18

-12

98

-96

2

Drag down 4.

12)(18

-12

98

-96

24

Finally, ascertain how many 12s there are in 24? The answer is 2: 12+12=24. Multiply 12 by 2: the result is

24. Subtract 24 from 24: the result is 0.

12)(182

-12

98

-96

24

24

0

As we have obtained zero and there are no more numbers to divide, our work is done.

So, $2184 \div 12 = 182$.

Now, it's time to attempt the following problems by yourself:

Exercise 1, Find $812 \div 4$.

Answer: 203.

Exercise 2, Find $93 \div 3$.

Answer: 31.

Exercise 3, Find $255 \div 5$.

Answer: 51.

Exercise 4, Find $1792 \div 32$.

Answer: 56.

Exercise 5, Find $34272 \div 224$.

Answer: 153.

Division with remainder

When we divided whole numbers, we worked with examples where the result is a whole number. However, it is not true in general.

For example, what if we try to divide 11 by 4? You can see that there are two 4s in 11 and something extra: $11=4+4+3$ or $11=4\times2+3$.

This procedure is called division with remainder. It takes place when the result of division is not a whole number.

In general, if m is a dividend, n is a divisor, q is a quotient and r is a remainder, then $m=n\times q+r$. Here, keep in mind that n should be greater than r and all the numbers should be whole numbers.

In the above example, we could write $15=7\times1+8$. Although it is correct, $7<8$ so the remainder is found incorrectly.

When the remainder equals 0, the result of division is a whole number.

For example, since $15\div3=5$, we can write that $15=5\times3+0$ or simply $15=5\times3$.

In general, we can obtain the remainder using the same technique that we saw while dividing whole numbers.

Let's try out a few examples to understand this concept better.

Example 1, Find the remainder after the division of 87 by 2.

First, write the numbers in special form:

2)

Now let's proceed to divide 8 by 2. You can ascertain that there are four 2s are in 8: $2+2+2+2=8$. Write down 4 and multiply it by 2: the result is 8. Write it down.

2)(4

-8

Now, subtract 8 from 8: the result is 0.

2)(4

-8

0

Drag down 7.

2)(4

-8

07

Here, we have first carried out division followed by multiplication and subtraction.

Let's proceed in the same way and divide 7 by 2. In this case, there are three 2s and an additional 1: 7=2×3+1.

Now, multiply 2 and 3: the result is 6.

2)(43

-8

07

-6

Now, subtract 7 from 6: the result is 1.

2)(43

-8

07

-6

1

Since 1<2, i.e. the remainder is less than the divisor, we are done with the problem.

Thus, the remainder is 1: 87=2×43+1.

Let's look at another example.

Example 2, Find the remainder after the division of 74 by 3.

Again, write the two numbers in special form:

3)(

Think: how many 3s are there in 7? 7=3+3+1. Clearly, there are two 3s and something extra in 7.

Write down 2 and multiply it by 3: the result is 6.

3)(2

-6

After that, subtract 6 from 7: the result is 1.

3)(2

-6

1

Drag 4 down.

3)(2

-6_

14

Now continue with 14: how many 3s are there in 14? You can see that there are four 3s and something extra: 14=3+3+3+3+2.

Multiply 3 and 4: the result is 12.

3)(24

-6_

14

12

Subtract 14 from 12: the result is 2.

3)(24

-6_

14

12

2

We are done, because we have gotten a number that is less than 3 (the divisor).

Thus, the remainder is 2. 74=3×24+2.

Finally, let's work with a slightly harder example.

Example 3, Find the remainder after the division of 2194 by 12.

As usual, write down both numbers in special form:

12)

Think: how many 12s are in 2? The answer is zero: after all, 12 is greater than 2.

Thus, we need to add the next digit as well (take 21 instead of just 2). Now, how many 12s are there in 21? Clearly, there is one 12 in there: 12+9=21.

Write down 1 and multiply it by 12: the result is 12. Write it down.

12)(1

-12

Subtract 12 from 21: the result is 9.

12)(1

-12

9

Drag down the other 9.

12)(1

-12_

99

Continue with 99: how many 12s are there in 99? You can see that there are eight 12s: 99=12+12+12+12+12+12+12+12

Multiply 12 and 8: the result is 96.

12)(18

-12_

99

-96

Subtract 99 from 96: the result is 3.

12)(18

-12_

99

-96

3

Drag down 4.

12)(18

-12_

99

-96

34

Finally, we must find out how many 12s there are in 34. You can determine that there are two 12s: 24=12+12+10.

Multiply 12 by 2: the result is 24.

Subtract 34 from 24: the result is 10.

12)(182

-12_

99

-96

34

-24

10

Since 10<12, we are finished with the problem.

Thus, the remainder is 10: $2184=12\times182+10$

Now go ahead and solve the following problems by yourself:

Exercise 1, Find the remainder after the division of 85 by 2.

Answer: 1 ($85=2\times42+1$).

Exercise 2, Find the remainder after the division of 98 by 3.

Answer: 2 ($98=3\times32+2$).

Exercise 3, Find the remainder after the division of 160 by 4.

Answer: 0 ($160=4\times40$).

Exercise 4, Find the remainder after the division of 1250 by 16.

Answer: 2 ($1250=16\times78+2$).

Exercise 5, Find the remainder after the division of 57130 by 342.

Answer: 16 ($57130=342\times167+16$).

Practice Questions

Work each question before reading its solution. The set runs from direct recall and substitution to the applied questions that exams actually use to separate grades.

Q1. Which of these are whole numbers: 7, 0, -3, $\frac{1}{2}$, 4.0, $\sqrt{9}$?

SOLUTION

7, 0, 4.0, and $\sqrt{9} = 3$ are whole numbers; -3 is an integer but not whole, and $\frac{1}{2}$ is neither. Whole numbers are $0, 1, 2, 3, \ldots$: the counting numbers plus zero, no negatives, no fractions. A disguised value counts by what it equals, not how it is written.

Q2. What is the place value of each digit in 47,062?

SOLUTION

4 ten-thousands (40,000), 7 thousands (7,000), 0 hundreds, 6 tens (60), 2 ones. The zero is a place-holder doing real work: without it the number collapses to 4,762. Place value is why 10 digits suffice for every number.

Q3. Evaluate $48 \div 6 \times 2 + 3^2 - (4 + 1)$.

SOLUTION

Parentheses: 5. Exponent: 9. Then division and multiplication left to right: $48 \div 6 = 8$, $8 \times 2 = 16$. Then $16 + 9 - 5 = 20$. Division and multiplication share one rank read left to right; treating "multiplication first" as absolute is the classic error that would give 4 instead of 16.

Q4. Name the property: (a) $23 + 58 = 58 + 23$, (b) $(7 \times 5) \times 2 = 7 \times (5 \times 2)$, (c) $6 \times (10 + 3) = 60 + 18$.

SOLUTION

(a) Commutativity of addition. (b) Associativity of multiplication. (c) Distributivity of multiplication over addition. These 3 licenses are why mental math works: you may reorder, regroup, and split freely.

Q5. Compute 17×98 mentally using a property.

SOLUTION

$17 \times 98 = 17 \times (100 - 2) = 1700 - 34 = 1666$. Distributivity converts an awkward multiplication into a round one and a small correction. The same trick handles 99, 102, 4.99: multiply by the round neighbor, then adjust.

Q6. Is subtraction commutative or associative on whole numbers? Show with counterexamples.

SOLUTION

Neither: $5-3 = 2$ but $3-5$ is not even a whole number, and $(8-4)-2 = 2$ while $8-(4-2) = 6$. Subtraction also fails closure: whole numbers are closed under addition and multiplication only. This is exactly why integers were invented.

Q7. What are the identity elements for addition and multiplication, and what does the identity property say?

SOLUTION

Zero for addition ($n+0 = n$) and 1 for multiplication ($n\times1 = n$): the elements that change nothing. Related but distinct: multiplying by 0 destroys everything to 0, which is a separate property, not an identity.

Q8. A school orders 24 boxes of 36 pencils and distributes 15 pencils to each of 52 classes. How many pencils remain?

SOLUTION

Total: $24 \times 36 = 864$. Distributed: $15 \times 52 = 780$. Remaining: $864 - 780 = 84$. Two multiplications and a subtraction, organized by writing the plan before the arithmetic.

Q9. Round 738,499 to the nearest thousand, and explain why the answer is not 739,000.

SOLUTION

The thousands digit is 8; the decider is the hundreds digit, 4, which rounds down: 738,000. The trailing 99 tempts an upward cascade, but rounding looks at exactly one digit, the one below the target place. 738,499 sits closer to 738,000 than to 739,000 by 501 to 499... check: distance down 499, distance up 501. Down wins.

Q10. Every whole number is either even or odd. Prove that the sum of any 2 odd numbers is even.

SOLUTION

Odd numbers have the form $2m + 1$ and $2n + 1$. Their sum is $2m + 2n + 2 = 2(m + n + 1)$, visibly twice a whole number, hence even. Parity arguments like this are the first taste of algebraic proof: 2 letters replace infinitely many checks.

Answer Key

QUICK ANSWERS

Q1 7, 0, 4.0, $\sqrt{9}$ **Q2** 40,000 + 7,000 + 0 + 60 + 2 **Q3** 20 **Q4** commutative, associative, distributive **Q5** 1666

Q6 no and no; $3 - 5$ escapes the whole numbers **Q7** 0 and 1 **Q8** 84 **Q9** 738,000 **Q10** $(2m + 1) + (2n + 1) = 2(m + n + 1)$

Revision Sheet

The set

Whole numbers: $0, 1, 2, 3, \ldots$. Counting numbers plus zero. No negatives, no fractions.

Identities

$n + 0 = n$; $n \times 1 = n$. And separately, $n \times 0 = 0$.

Closure

Closed under + and $\times$; NOT under $-$ or $\div$. That failure motivates integers and fractions.

Order of operations

Brackets, exponents, then $\times/\div$ left to right, then $+/-$ left to right.

The 3 licenses

Commute (reorder), associate (regroup), distribute (split). All mental math lives here.

Rounding

Look at exactly one digit below the target place: 5 or more rounds up.

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