TRIGONOMETRY

Trigonometric Identities

The identities that matter, how they derive from one another, proof technique, and a 10-question practice set with full solutions.

SUBJECT	LEVEL	INCLUDES
Mathematics	High school and early college	Practice set, answer key, revision sheet

Read this note online, with updates: gauravtiwari.org/study-notes/trigonometric-identities

INSIDE

Trigonometric identities are equations involving trig functions that hold true for every valid value of the variable. You'll use them constantly, whether you're simplifying expressions, solving equations, or proving other results. They're the backbone of trigonometry.

Sine, cosine, and tangent are your primary trig functions. Secant, cosecant, and cotangent round out the set. All six build on each other, and the identities below connect them in ways you need to know cold.

What are Trigonometric Identities?

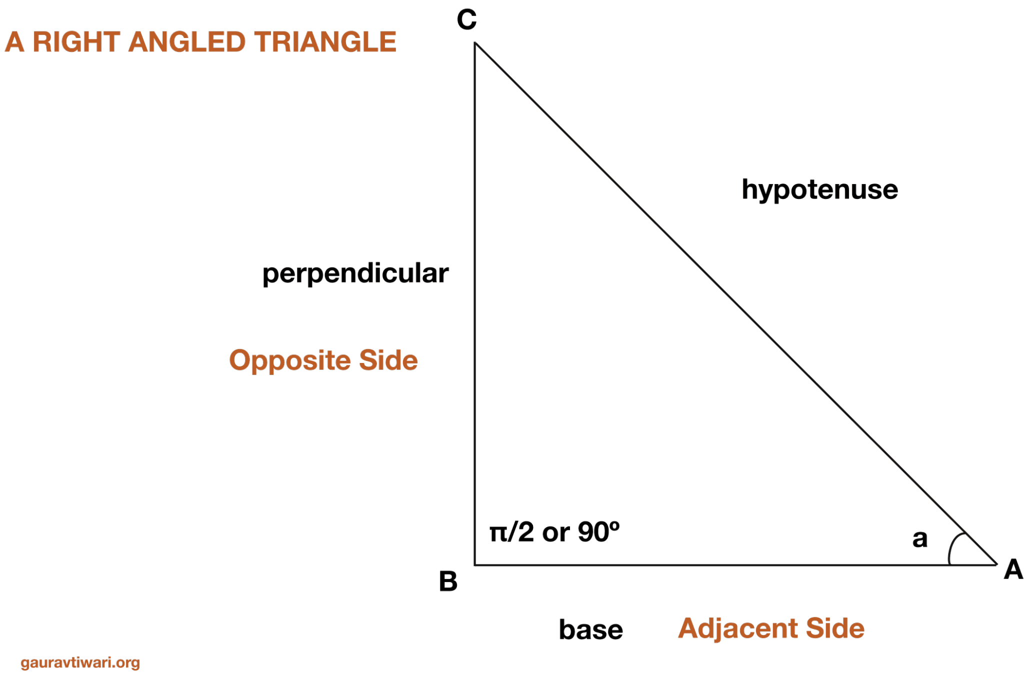
A trigonometric identity is an equation involving trig functions that's true for all values of the variable where both sides are defined. This isn't something you verify with one angle and call it done. It has to work everywhere.

There are many distinct identities, and they involve both the side lengths and angles of a triangle. One thing to keep in mind: most of the basic definitions start with a right-angled triangle. If you need a refresher on how functions behave at their core, check out the notes on function notations and rules.

All trigonometric identities trace back to six ratios: sine, cosine, tangent, cosecant, secant, and cotangent. Each ratio is defined using the sides of a right triangle (adjacent, opposite, and hypotenuse). Once you understand these six, every identity below is just algebra.

Trigonometric Identities

Using a right-angled triangle as your reference, here's how each trig function is defined.



Most Trigonometric Identities are defined on a Right Angled Triangle

$$\sin\theta = \frac{\text{Opposite side (Perpendicular)}}{\text{Hypotenuse}}$$

$$\cos\theta = \frac{\text{Adjacent side(Base)}}{\text{Hypotenuse}}$$

$$\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent side}}$$

$$\text{cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Opposite side}}$$

$$\cot\theta = \frac{\text{Adjacent side}}{\text{Opposite side}}$$

Ratio Trigonometric Identities

These two identities express tangent and cotangent in terms of sine and cosine. You'll reach for these constantly when simplifying expressions.

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

Each trig function has a reciprocal partner. Memorize these pairs and you'll move through problems much faster.

$\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

$\sec\ \theta = \dfrac{1}{\cos\theta}$

$\cot\ \theta = \dfrac{1}{\tan\theta}$

$\sin\ \theta = \dfrac{1}{\operatorname{cosec}\theta}$

$\cos\ \theta = \dfrac{1}{\sec\theta}$

$\tan\ \theta = \dfrac{1}{\cot\theta}$

All of these come directly from the right-angled triangle definitions. When you know the height and base of a right triangle, you can find every trig value. The reciprocal identities are just the original ratios flipped.

Periodicity Identities (in Radians)

These are also called co-function identities. They let you shift angles by $\pi/2$, π, 2π, and so on. You'll find them especially useful when working with the unit circle.

1. $\sin(\pi/2 - A) = \cos A$ and $\cos(\pi/2 - A) = \sin A$
2. $\sin(\pi/2 + A) = \cos A$ and $\cos(\pi/2 + A) = -\sin A$
3. $\sin(3\pi/2 - A) = -\cos A$ and $\cos(3\pi/2 - A) = -\sin A$
4. $\sin(3\pi/2 + A) = -\cos A$ and $\cos(3\pi/2 + A) = \sin A$
5. $\sin(\pi - A) = \sin A$ and $\cos(\pi - A) = -\cos A$
6. $\sin(\pi + A) = -\sin A$ and $\cos(\pi + A) = -\cos A$
7. $\sin(2\pi - A) = -\sin A$ and $\cos(2\pi - A) = \cos A$
8. $\sin(2\pi + A) = \sin A$ and $\cos(2\pi + A) = \cos A$

All trig functions are cyclic. They repeat after a fixed period. That period differs by function: sine and cosine repeat every 2π, while tangent and cotangent repeat every π.

Cofunction Identities (in Degrees)

If you prefer working in degrees, here are the same co-function relationships. Each pair swaps when you subtract from 90 degrees.

$\sin(90° - x) = \cos x$

$\cos(90° - x) = \sin x$

$\tan(90° - x) = \cot x$

$\cot(90° - x) = \tan x$

$\sec(90° - x) = \mathrm{cosec} x$

$\mathrm{cosec}(90° - x) = \sec x$

Sum and Difference Identities

These let you break apart the trig function of a sum or difference of two angles. You'll use them heavily in calculus and in proving other identities. If you're building toward calculus, a solid calculus textbook will show you exactly how these identities power derivative and integral formulas.

$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$

$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

$\tan(x + y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

$\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x - y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

REMEMBER

The sum and difference identities are the foundation for deriving both double angle and half angle formulas. Master these first, and the rest follow naturally.

Double Angle Identities

Double angle formulas give you the trig values of $2x$ in terms of x. These show up everywhere, from integration to signal processing.

$$\sin 2x = 2\sin x\cos x = \frac{2\tan x}{1+\tan^2 x}$$

$$\cos 2x = \cos^2 x - \sin^2 x = \frac{1-\tan^2 x}{1+\tan^2 x}$$

$$\cos(2x) = 2\cos^2(x) - 1 = 1 \text{ to } 2\sin^2(x)$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$\sec 2x = \frac{\sec^2 x}{2-\sec^2 x}$$

$$\text{cosec}2x = \frac{1}{2}\sec x\,\text{cosec}x$$

Triple Angle Identities

These express trig functions of $3x$ in terms of x. They're less common than double angle formulas but still worth knowing for competitive exams and advanced problem solving.

$$\sin 3x = 3\sin x - 4\sin^3 x$$

$$\cos 3x = 4\cos^3 x - 3\cos x$$

$$\tan 3x = \frac{3\tan x - \tan^3 x}{1-3\tan^2 x}$$

Half Angle Identities

Half angle identities let you find trig values of $x/2$ when you know the value at x. The plus-or-minus sign depends on the quadrant where $x/2$ falls.

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\tan\frac{x}{2} = \sqrt{\frac{1-\cos x}{1+\cos x}} = \frac{1-\cos x}{\sin x}$$

Product Identities

Product-to-sum identities convert a product of two trig functions into a sum. These are particularly handy when you're integrating products of sines and cosines.

$$\sin x\cos y = \frac{\sin(x+y)+\sin(x-y)}{2}$$

$$\cos x\cos y = \frac{\cos(x+y)+\cos(x-y)}{2}$$

$$\sin x\sin y = \frac{\cos(x-y)-\cos(x+y)}{2}$$

Sum to Product Identities

These work the other way: they convert a sum or difference of trig functions into a product. You'll find them useful for solving equations where you need to factor expressions.

$$\sin x+\sin y = 2\sin\frac{(x+y)}{2}\cos\frac{(x-y)}{2}$$

$$\sin x-\sin y = 2\cos\frac{(x+y)}{2}\sin\frac{(x-y)}{2}$$

$$\cos x+\cos y = 2\cos\frac{(x+y)}{2}\cos\frac{(x-y)}{2}$$

$\cos x - \cos y = -2\sin\frac{(x+y)}{2}\sin\frac{(x-y)}{2}$

Pythagorean Trigonometric Identities

These three identities come directly from the Pythagorean theorem. They're probably the most used trigonometric identities in all of mathematics, so make sure you know them by heart.

$\sin^2 a + \cos^2 a = 1$

$1 + \tan^2 a = \sec^2 a$

$\text{cosec}^2 a = 1 + \cot^2 a$

KEY INSIGHT

The Pythagorean identities aren't separate facts to memorize. If you know $\sin^2 a + \cos^2 a = 1$, you can derive the other two by dividing both sides by $\cos^2 a$ or $\sin^2 a$. That's one identity, not three.

Trigonometric Identities of Opposite Angles

When you negate an angle, some trig functions flip sign and others don't. Cosine and secant are even functions (they stay the same), while sine, tangent, cotangent, and cosecant are odd functions (they flip sign).

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

$\cot(-\theta) = -\cot\theta$

$\sec(-\theta) = \sec\theta$

$\text{cosec}\,(-\theta) = -\text{cosec}\,\theta$

Understanding even and odd trig functions is essential when you're evaluating expressions at specific angles. For a deeper look at how functions behave at boundary values, see the notes on zero of a function.

Trigonometric Identities of Complementary Angles

Two angles are complementary when they add up to 90 degrees. The trig function of one equals the co-function of the other. This is actually where the "co" in cosine, cotangent, and cosecant comes from.

$\sin(90° - \theta) = \cos\theta$

$\cos(90° - \theta) = \sin\theta$

$\tan(90° - \theta) = \cot\theta$

$\cot(90° - \theta) = \tan\theta$

$\sec(90° - \theta) = \operatorname{cosec}\theta$

$\operatorname{cosec}(90° - \theta) = \sec\theta$

Trigonometric Identities of Supplementary Angles

Two angles are supplementary when they add up to 180 degrees. Here's how each trig function behaves when you subtract your angle from 180 degrees.

$\sin(180° - \theta) = \sin\theta$

$\cos(180° - \theta) = -\cos\theta$

$\operatorname{cosec}(180° - \theta) = \operatorname{cosec}\theta$

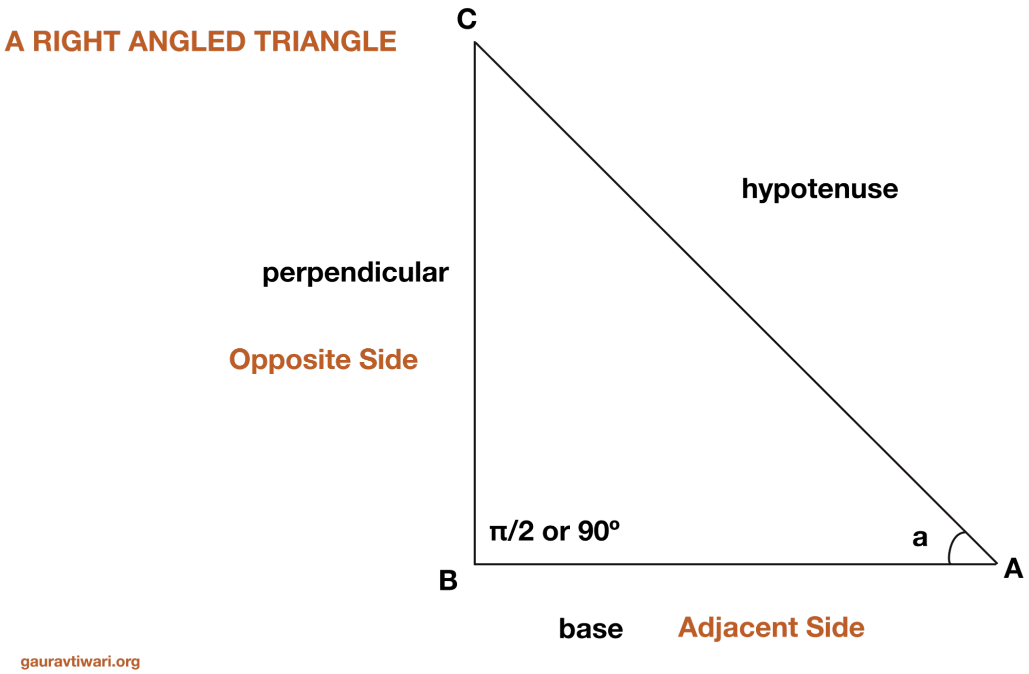
$\sec(180° - \theta) = -\sec\theta$

$\tan(180° - \theta) = -\tan\theta$

$\cot(180° - \theta) = -\cot\theta$

Proofs of Trigonometric Identities

Now let's prove the three Pythagorean identities. These aren't arbitrary formulas. They follow directly from the Pythagorean theorem applied to a right-angled triangle. Consider a right-angled $\triangle ABC$, which is right-angled at B.



Most Trigonometric Identities are defined on a Right Angled Triangle

Applying the Pythagorean theorem to this triangle, you get:

$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$

$AC^2 = AB^2 + BC^2 \ldots (1)$

Trigonometric Identity 1

Divide each term of equation (1) by AC^2:

$$\frac{AC^2}{AC^2} = \frac{AB^2}{AC^2} + \frac{BC^2}{AC^2}$$

$$\implies \frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} = 1$$

$$\implies (\frac{AB}{AC})^2 + (\frac{BC}{AC})^2 = 1 \ldots (2)$$

You know that $\frac{AB}{AC}$ is $\cos a$ and $\frac{BC}{AC}$ is $\sin a$.

Substituting into equation (2):

$\sin^2 a + \cos^2 a = 1$

This identity is valid for angles $0 \leq a \leq 90°$.

Trigonometric Identity 2

Now divide equation (1) by AB^2:

$$\frac{AC^2}{AB^2} = \frac{AB^2}{AB^2} + \frac{BC^2}{AB^2}$$
$$\implies \frac{AC^2}{AB^2} = 1 + \frac{BC^2}{AB^2}$$
$$\implies (\frac{AC}{AB})^2 = 1 + (\frac{BC}{AB})^2 \ldots (3)$$

Here, $\frac{AC}{AB}$ is the inverse of $\cos a$, which is $\sec a$, and $\frac{BC}{AB}$ is $\tan a$.

Replacing the values in equation (3):

$1 + \tan^2 a = \sec^2 a \ldots (4)$

Since $\tan a$ is not defined for $a = 90°$, this identity holds true for $0 \leq a < 90°$.

Trigonometric Identity 3

Similarly, dividing equation (1) by BC^2 gives you:

$\operatorname{cosec}^2 a = 1 + \cot^2 a \ldots (5)$

Since $\operatorname{cosec} a$ and $\cot a$ are not defined for $a = 0°$, this identity holds for all values where $0° < a \leq 90°$.

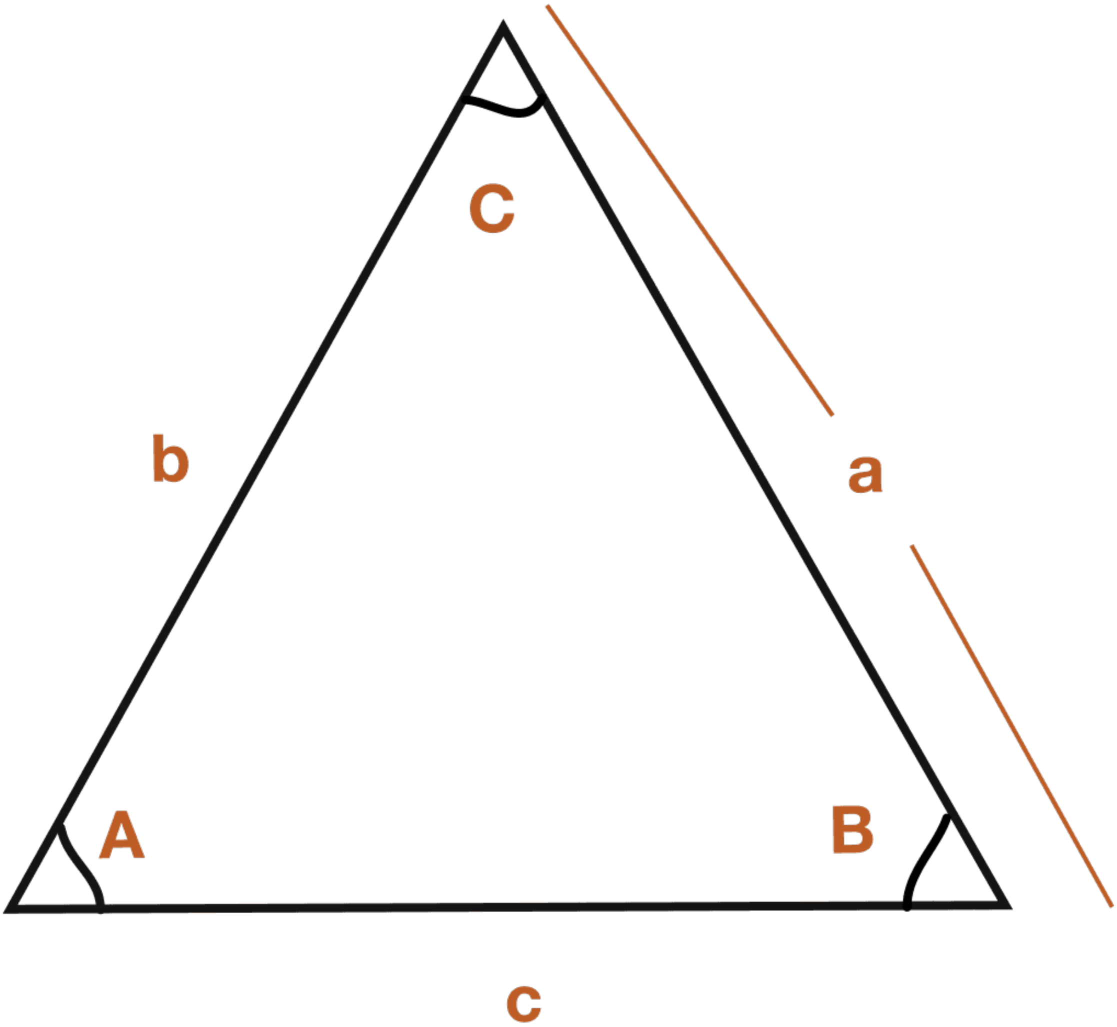
Triangle Identities (Sine, Cosine, and Tangent Rules)

The identities above apply specifically to right-angled triangles. But there are three rules that work for any triangle at all. These are the sine rule, cosine rule, and tangent rule. You'll find these trigonometric identities essential in any course that covers triangle geometry. For a collection of time-saving mathematics formulas and theorems, including these laws, bookmark that reference.

- Sine law
- Cosine law
- Tangent law

Also see: Triangle Inequality

Sine Law



Triangle identities (the Sine, Cosine, and Tangent rules)

If A, B and C are the vertices of a triangle, and a, b and c are the opposite sides respectively, the sine rule states:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or equivalently, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Cosine Law

The cosine rule generalizes the Pythagorean theorem to any triangle. When angle C is 90 degrees, the $\cos C$ term drops to zero and you get the standard Pythagorean theorem back.

$$c^2 = a^2 + b^2 - 2ab\cos C$$

or, $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

Practice Questions

Work each question before reading its solution. The set runs from direct recall and substitution to the applied questions that exams actually use to separate grades. All 10 also appear in the downloadable PDF with a separate answer key.

Question 1. From $\sin^2\theta + \cos^2\theta = 1$, derive the other 2 Pythagorean identities.

Solution. Divide by $\cos^2\theta$: $\tan^2\theta + 1 = \sec^2\theta$. Divide by $\sin^2\theta$: $1 + \cot^2\theta = \csc^2\theta$. One identity, 2 divisions, the whole Pythagorean family. They are the unit circle's equation $x^2 + y^2 = 1$ in disguise.

Question 2. Given $\sin\theta = \frac{3}{5}$ with θ in the second quadrant, find $\cos\theta$ and $\tan\theta$.

Solution. $\cos^2\theta = 1 - \frac{9}{25} = \frac{16}{25}$, and in quadrant II cosine is negative: $\cos\theta = -\frac{4}{5}$. Then $\tan\theta = \frac{3/5}{-4/5} = -\frac{3}{4}$. The identity gives the magnitude; the quadrant gives the sign. Forgetting the sign check is the classic dropped mark.

Question 3. Compute $\sin 75°$ exactly using a sum formula.

Solution. $\sin 75° = \sin(45° + 30°) = \sin 45\cos 30 + \cos 45\sin 30 = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$. The sum formulas turn the 2 memorized triangles into exact values for a whole family of angles.

Question 4. Derive the double-angle formula for sine, and state the 3 forms for cosine.

Solution. Set $b = a$ in $\sin(a+b)$: $\sin 2a = 2\sin a\cos a$. Similarly $\cos 2a = \cos^2 a - \sin^2 a$, and substituting $\sin^2 = 1 - \cos^2$ or the reverse gives $2\cos^2 a - 1$ and $1 - 2\sin^2 a$. Choose the form that eliminates what you don't know.

Question 5. Given $\cos\theta = \frac{1}{3}$, find $\cos 2\theta$ without finding θ.

Solution. $\cos 2\theta = 2\cos^2\theta - 1 = 2\cdot\frac{1}{9} - 1 = -\frac{7}{9}$. No inverse-trig, no calculator: the right identity form answers directly from the given data.

Question 6. Prove the identity $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta} = 2\csc\theta$.

Solution. Combine over the common denominator: numerator $= \sin^2\theta + (1+\cos\theta)^2 = \sin^2 + 1 + 2\cos + \cos^2 = 2 + 2\cos\theta$. So the sum is $\dfrac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)} = \dfrac{2}{\sin\theta} = 2\csc\theta$. Standard proof craft: work one side, find common denominators, and let $\sin^2 + \cos^2 = 1$ collapse the mess.

Question 7. Solve $\sin 2x = \cos x$ for $0 \le x < 2\pi$.

Solution. Rewrite: $2\sin x\cos x - \cos x = 0$, so $\cos x(2\sin x - 1) = 0$. Either $\cos x = 0$: $x = \frac{\pi}{2}, \frac{3\pi}{2}$; or $\sin x = \frac{1}{2}$: $x = \frac{\pi}{6}, \frac{5\pi}{6}$. Four solutions. Factoring, never dividing by $\cos x$, preserves the solutions division would delete.

Question 8. Express $\sin^2\theta$ without a square, and name the application this serves.

Solution. From $\cos 2\theta = 1 - 2\sin^2\theta$: $\sin^2\theta = \frac{1-\cos 2\theta}{2}$. This power-reduction step is what makes $\int \sin^2\theta\, d\theta$ integrable in first-year calculus, and it reappears in signal processing as the DC component of a squared sinusoid.

Question 9. A wave $y = 3\sin\omega t + 4\cos\omega t$ is a single sinusoid in disguise. Find its amplitude and the identity behind the rewrite.

Solution. $a\sin x + b\cos x = R\sin(x+\phi)$ with $R = \sqrt{a^2+b^2}$ and $\tan\phi = b/a$. Here $R = \sqrt{9+16} = 5$: amplitude 5, phase $\phi = \arctan\frac{4}{3} \approx 53°$. Adding same-frequency waves never changes the frequency, only amplitude and phase: the working heart of AC circuit analysis.

Question 10. Verify numerically that tan has period π while sin needs 2π, and explain from the definitions.

Solution. $\tan(\theta+\pi) = \frac{\sin(\theta+\pi)}{\cos(\theta+\pi)} = \frac{-\sin\theta}{-\cos\theta} = \tan\theta$: both components flip sign and the ratio survives. Sine alone flips: $\sin(\theta+\pi) = -\sin\theta$, so it needs the full 2π to return. Check: $\tan 30° = \tan 210° \approx 0.577$, while $\sin 210° = -\frac{1}{2} \ne \sin 30°$.

Tangent Law

The tangent rule (also called the law of tangents) relates the sides and angles of any triangle:

$$\frac{a-b}{a+b} = \frac{\tan\frac{A-B}{2}}{\tan\frac{A+B}{2}}$$

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Q1. From $\sin^2\theta + \cos^2\theta = 1$, derive the other 2 Pythagorean identities.

SOLUTION

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Q7. Solve $\sin 2x = \cos x$ for $0 \leq x < 2\pi$.

SOLUTION

Rewrite: $2\sin x\cos x - \cos x = 0$, so $\cos x(2\sin x - 1) = 0$. Either $\cos x = 0$: $x = \frac{\pi}{2}, \frac{3\pi}{2}$; or $\sin x = \frac{1}{2}$: $x = \frac{\pi}{6}, \frac{5\pi}{6}$. Four solutions. Factoring, never dividing by $\cos x$, preserves the solutions division would delete.

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Answer Key

QUICK ANSWERS

Q1 $\tan^2 +1 = \sec^2$; $1 + \cot^2 = \csc^2$ **Q2** $-\frac{4}{5}$ and $-\frac{3}{4}$ **Q3** $\frac{\sqrt{6}+\sqrt{2}}{4}$ **Q4** $2\sin a\cos a$; 3 cosine forms
Q5 $-\frac{7}{9}$
Q6 combine fractions; Pythagorean collapse **Q7** $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$ **Q8** $\frac{1-\cos 2\theta}{2}$; integrating even powers
Q9 amplitude 5, phase $\approx 53°$ **Q10** signs cancel in the ratio; period π

Revision Sheet

The master identity

$\sin^2\theta + \cos^2\theta = 1$; divided: $\tan^2 +1 = \sec^2$, $1 + \cot^2 = \csc^2$.

Sum formulas

$\sin(a \pm b) = \sin a\cos b \pm \cos a\sin b$; $\cos(a \pm b) = \cos a\cos b \mp \sin a\sin b$.

Double angle

$\sin 2a = 2\sin a\cos a$; $\cos 2a = \cos^2 - \sin^2 = 2\cos^2 -1 = 1 - 2\sin^2$.

Power reduction

$\sin^2 a = \frac{1-\cos 2a}{2}$, $\cos^2 a = \frac{1+\cos 2a}{2}$.

Wave addition

$a\sin x + b\cos x = \sqrt{a^2 + b^2}\,\sin(x + \phi)$.

Proof craft

Work the messier side, convert to sin and cos, combine fractions, factor instead of dividing.

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