FLUID MECHANICS

Pascal's Law

Pressure transmission in fluids, the hydraulic multiplication trick and its honest cost, and a 10-question practice set with full solutions.

SUBJECT	LEVEL	INCLUDES
Physics	High school and early college	Practice set, answer key, revision sheet

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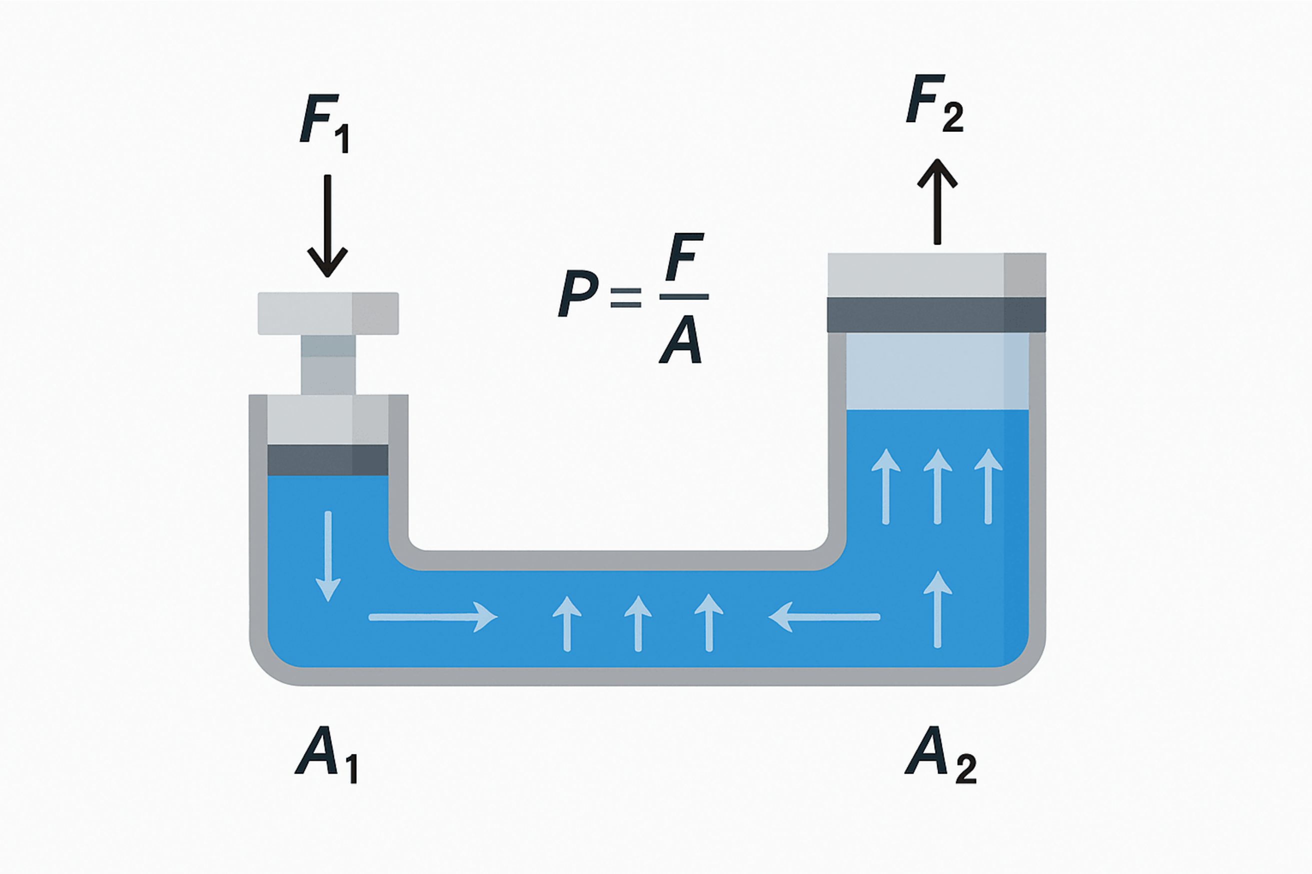
INSIDE

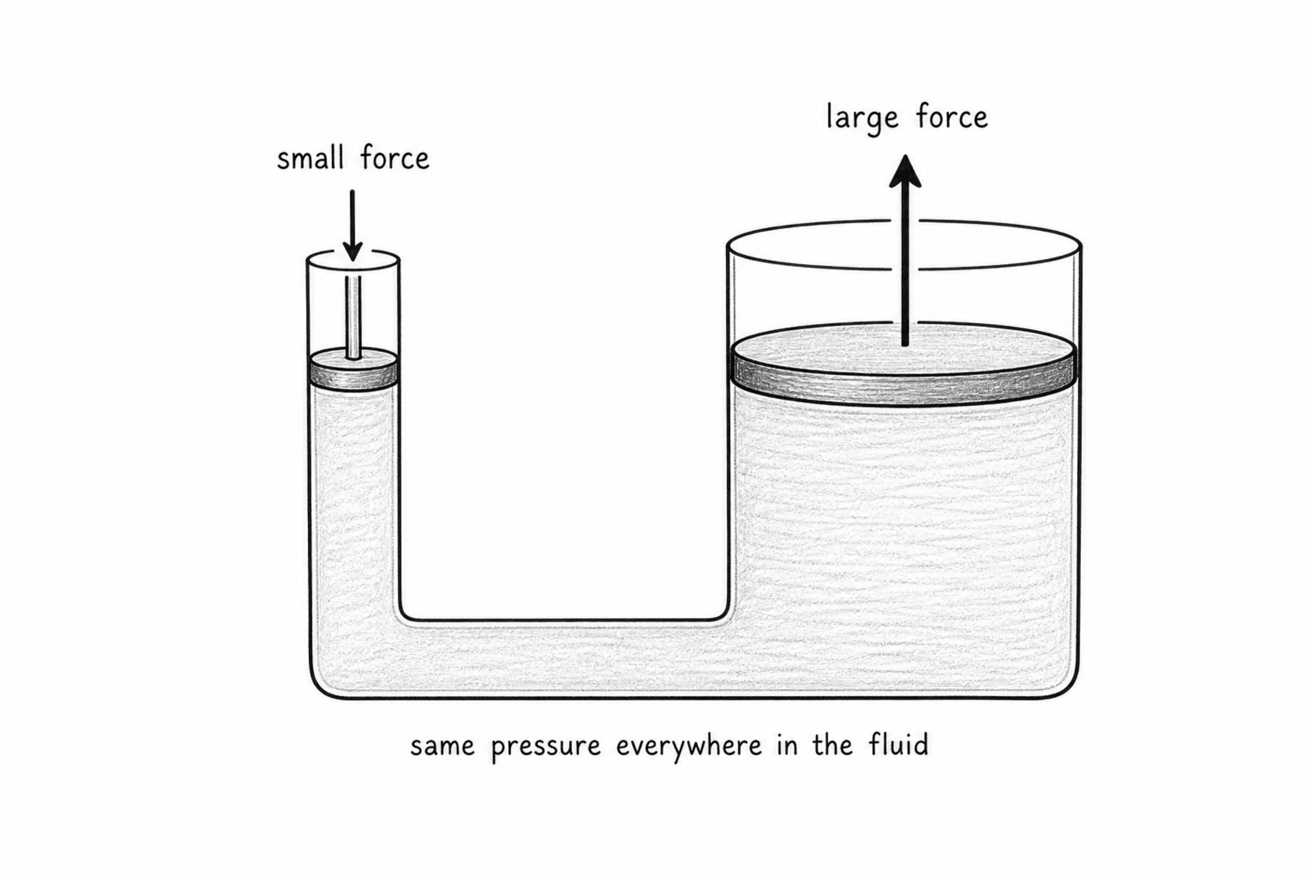
Pascal's law states that any pressure applied to a confined, incompressible fluid is transmitted equally and undiminished to every point in the fluid and to the walls of its container. Formulated by Blaise Pascal in the 1650s, the principle is the foundation of every hydraulic system in use today, car brakes, aircraft controls, construction equipment, dental chairs, and the hydraulic press that turns a small force into a large one.



Pascal's law: pressure applied to an enclosed fluid is transmitted undiminished to every part of the fluid.

Statement and Formula

Pressure is force per unit area:



The hydraulic press: one pressure everywhere, so the wide piston turns a small force into a large one.

$$P = \frac{F}{A}$$

Pascal's law: in a confined incompressible fluid, the change in pressure at any point is transmitted equally throughout the fluid. So if a force F_1 is applied to area A_1, the pressure increase $P = F_1/A_1$ is felt everywhere. The same pressure acting on a different area A_2 produces a different force:

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \Rightarrow \quad F_2 = F_1 \cdot \frac{A_2}{A_1}$$

This is the working equation of every hydraulic system.

The Hydraulic Press

A small piston of area A_1 is connected by an oil-filled tube to a large piston of area A_2. Push down on the small piston with force F_1 and the large piston pushes up with force $F_2 = F_1 \cdot A_2/A_1$. If $A_2 = 100A_1$, a 100 N input produces 10,000 N of output force. That's mechanical advantage.

Conservation of Energy

Force amplification doesn't come free. The volume of fluid pushed out from under the small piston equals the volume pushed up under the large piston: $A_1 d_1 = A_2 d_2$. So the small piston moves a long distance d_1 for every tiny distance d_2 the large piston moves. The work done is the same: $F_1 d_1 = F_2 d_2$. Pascal's law trades distance for force, exactly what a lever does, only with fluid instead of a rigid bar.

Worked Example: Car Brakes

A driver pushes a brake pedal that drives a master cylinder piston with area $A_1 = 1.0\ \mathrm{cm}^2$. The hydraulic fluid connects to four wheel-cylinder pistons each with area $A_2 = 6.0\ \mathrm{cm}^2$. The driver applies $F_1 = 100\ \mathrm{N}$.

Pressure: $P = 100/(1.0 \times 10^{-4}) = 10^6$ Pa (about 10 atm). At each wheel cylinder: $F_2 = P \cdot A_2 = 10^6 \cdot 6.0 \times 10^{-4} = 600\ \mathrm{N}$. With four wheels: 2,400 N of total braking force, 24× the pedal force.

Pressure in a Fluid Column

For a fluid at rest in a gravitational field, the pressure at depth h below the surface is:

$$P = P_0 + \rho g h$$

where P_0 is the pressure at the surface (often atmospheric pressure, ~101 kPa). This is consistent with Pascal's law, any change in P_0 (the surface pressure) shifts the entire pressure profile uniformly. Pascal's law is the reason barometers work, why your ears pop on takeoff, and why dam walls are thicker at the bottom.

Applications

- **Car brakes.** Pedal force is converted to fluid pressure that pushes brake pads against wheel rotors at every wheel simultaneously.
- **Hydraulic jacks and presses.** Lift cars, crush scrap metal, stamp sheet steel, mould plastics, small input force, enormous output.
- **Aircraft controls.** Hydraulic systems move ailerons, rudders, elevators, landing gear, and flaps. Pilot inputs are amplified by hydraulics that can deliver megawatts of mechanical power.
- **Excavators and construction equipment.** Booms, buckets, and outriggers are all hydraulic.
- **Dental and barber chairs.** A pedal-operated hydraulic ram lifts an adult with light effort.
- **Toothpaste tube.** Squeeze anywhere, and Pascal's law sends pressure to the open end uniformly.

Related study notes: Archimedes' Principle, Pressure, Bernoulli's Principle, Density.

Practice Questions

Work each question before reading its solution. The set runs from direct recall and substitution to the applied questions that exams actually use to separate grades.

Q1. State Pascal's law.

SOLUTION

Pressure applied to an enclosed, incompressible fluid is transmitted undiminished to every point of the fluid and the container walls. Squeeze a sealed water bottle anywhere and the pressure rises by the same amount everywhere: the fluid has no way to favor a direction.

Q2. A hydraulic lift has a 2 cm^2 input piston and a 400 cm^2 output piston. What output force does a 100 N push produce?

SOLUTION

Equal pressure on both pistons: $\frac{F_1}{A_1} = \frac{F_2}{A_2}$, so $F_2 = 100 \times \frac{400}{2} = 20{,}000$ N, enough to hold a 2-tonne car. The area ratio is the whole machine.

Q3. That 200-fold force gain looks like free energy. Where is the catch?

SOLUTION

Volume must balance: $A_1 d_1 = A_2 d_2$, so the small piston travels 200 times farther than the load rises. Work in $= 100 \times d_1$ equals work out $= 20{,}000 \times d_1/200$. Force is multiplied, distance is divided, energy breaks even. Hydraulics trade stroke for strength.

Q4. Compute the pressure in the lift of Question 2, in pascals.

SOLUTION

$P = F_1/A_1 = 100/(2 \times 10^{-4}\ \text{m}^2) = 5 \times 10^5$ Pa, about 5 atmospheres. Both pistons feel this same pressure; only the areas differ.

Q5. Why must the working fluid be a liquid rather than a gas for stiff hydraulic response?

SOLUTION

Liquids are nearly incompressible: applied pressure transmits almost instantly and the pistons move in lockstep. A gas would first compress, absorbing the input stroke as spongy volume change before the load moves. Air in brake lines is dangerous for exactly this reason, and bleeding the brakes removes it.

Q6. Car brakes: you press the pedal with 50 N, the master cylinder piston is 1 cm^2, and each of the 4 caliper pistons is 5 cm^2. Find the force at each caliper (ignore the pedal's lever gain).

SOLUTION

Pressure $= 50/10^{-4} = 5 \times 10^5$ Pa. Each caliper: $F = PA = 5 \times 10^5 \times 5 \times 10^{-4} = 250$ N. One foot press becomes 1,000 N of total pad force before the pedal linkage even helps: Pascal's law is the brake system.

Q7. Does Pascal's law say pressure is the same at every depth in a fluid? Clarify.

SOLUTION

No. Gravity still adds ρgh with depth: a lake's bottom is at higher pressure than its surface. Pascal's law says a change applied to the fluid appears equally at all points, on top of the depth-dependent baseline. In compact hydraulic machines the height differences are negligible, so the distinction hides.

Q8. A sealed tube of toothpaste is squeezed at the bottom, yet paste exits at the top. Explain via Pascal.

SOLUTION

The squeeze raises pressure throughout the enclosed paste, which behaves as a thick fluid. The only yielding boundary is the nozzle opening, so the pressure pushes paste out there, regardless of where you squeezed. The same logic drives spray bottles and syringes.

Q9. A hydraulic press must lift 8,000 N using at most 200 N of input. What minimum area ratio is needed, and what is the distance cost?

SOLUTION

Ratio $\geq 8000/200 = 40$: the output piston needs at least 40 times the input area. The cost: the input piston moves 40 times the lift distance, so raising the load 5 cm takes 2 m of pumping, which is why real jacks use many short strokes with valves.

Q10. Blaise Pascal reportedly burst a barrel with a thin 10 m tube of water. Why does so little water wreck a barrel?

SOLUTION

Pressure depends on height, not volume: 10 m of water column adds $\rho g h \approx 1000 \times 9.8 \times 10 \approx 10^5$ Pa, a full atmosphere, to the barrel's interior. The barrel walls feel that pressure times their whole area, thousands of newtons, though the tube holds barely a glassful. Pressure cares about depth; force cares about area.

Answer Key

QUICK ANSWERS

Q1 applied pressure transmits undiminished everywhere **Q2** 20,000 N **Q3** the input travels 200 times farther; work balances **Q4** 5×10^5 Pa **Q5** liquids don't compress; gas would sponge the stroke **Q6** 250 N per caliper **Q7** no; changes transmit equally atop the ρgh gradient **Q8** pressure transmits to the nozzle, the only exit **Q9** 40:1; input strokes 40× the lift **Q10** ρgh gives $\sim$ 1 atm regardless of volume

Revision Sheet

The law

Applied pressure transmits undiminished through an enclosed fluid in all directions.

The machine equation

$\frac{F_1}{A_1} = \frac{F_2}{A_2}$: force scales with piston area.

No free energy

$A_1 d_1 = A_2 d_2$: multiplied force means divided distance. Work balances.

Why liquids

Nearly incompressible, so response is stiff and immediate. Air in the line = spongy brakes.

Depth still counts

Pascal adds changes on top of the static $P = P_0 + \rho gh$ gradient.

Applications

Car brakes, hydraulic jacks and presses, excavator arms, dental chairs, toothpaste tubes.

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