MECHANICS

Kinetic Energy

The formula and its square, the work-energy theorem, real comparisons, and a 10-question practice set with full solutions.

SUBJECT	LEVEL	INCLUDES
Physics	High school and early college	Practice set, answer key, revision sheet

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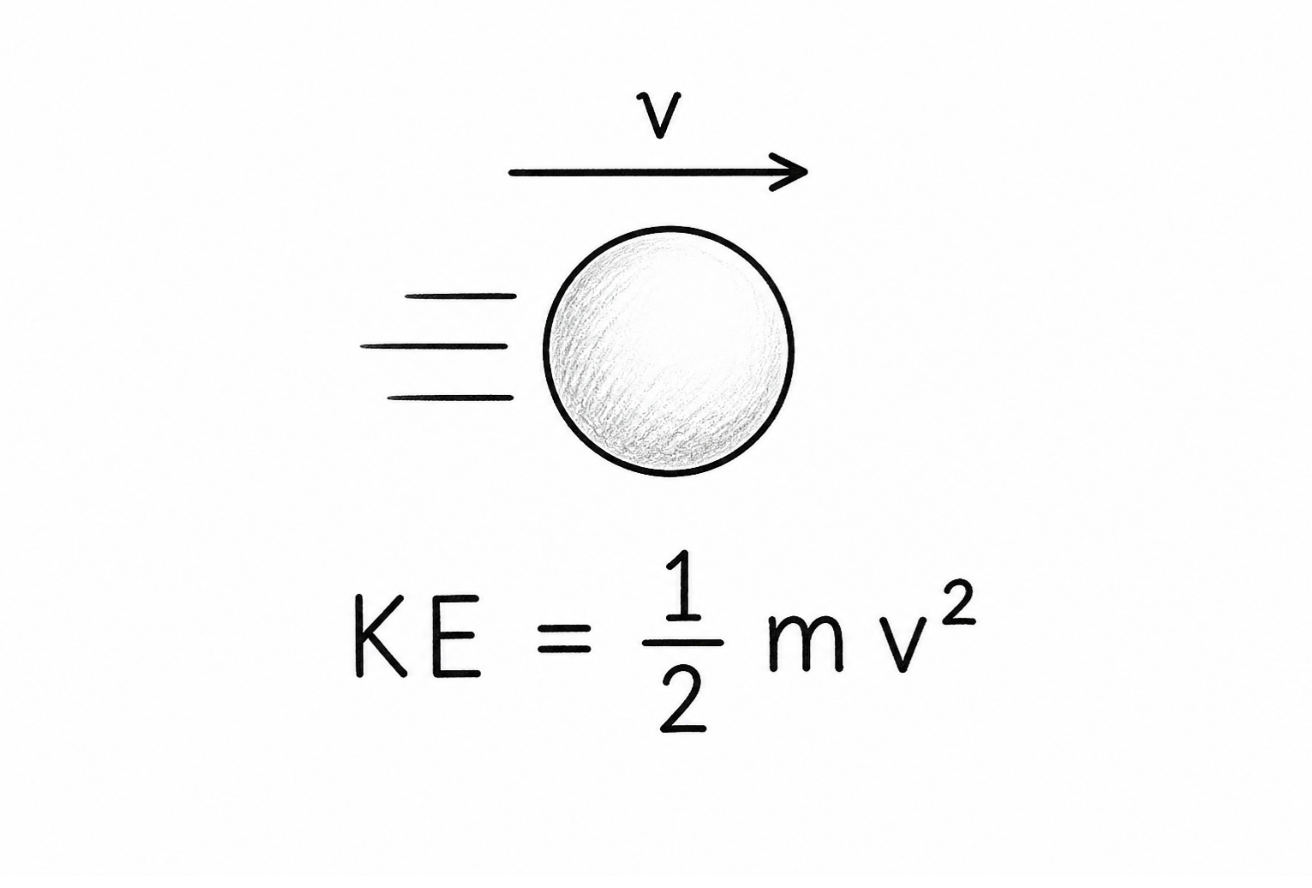
INSIDE

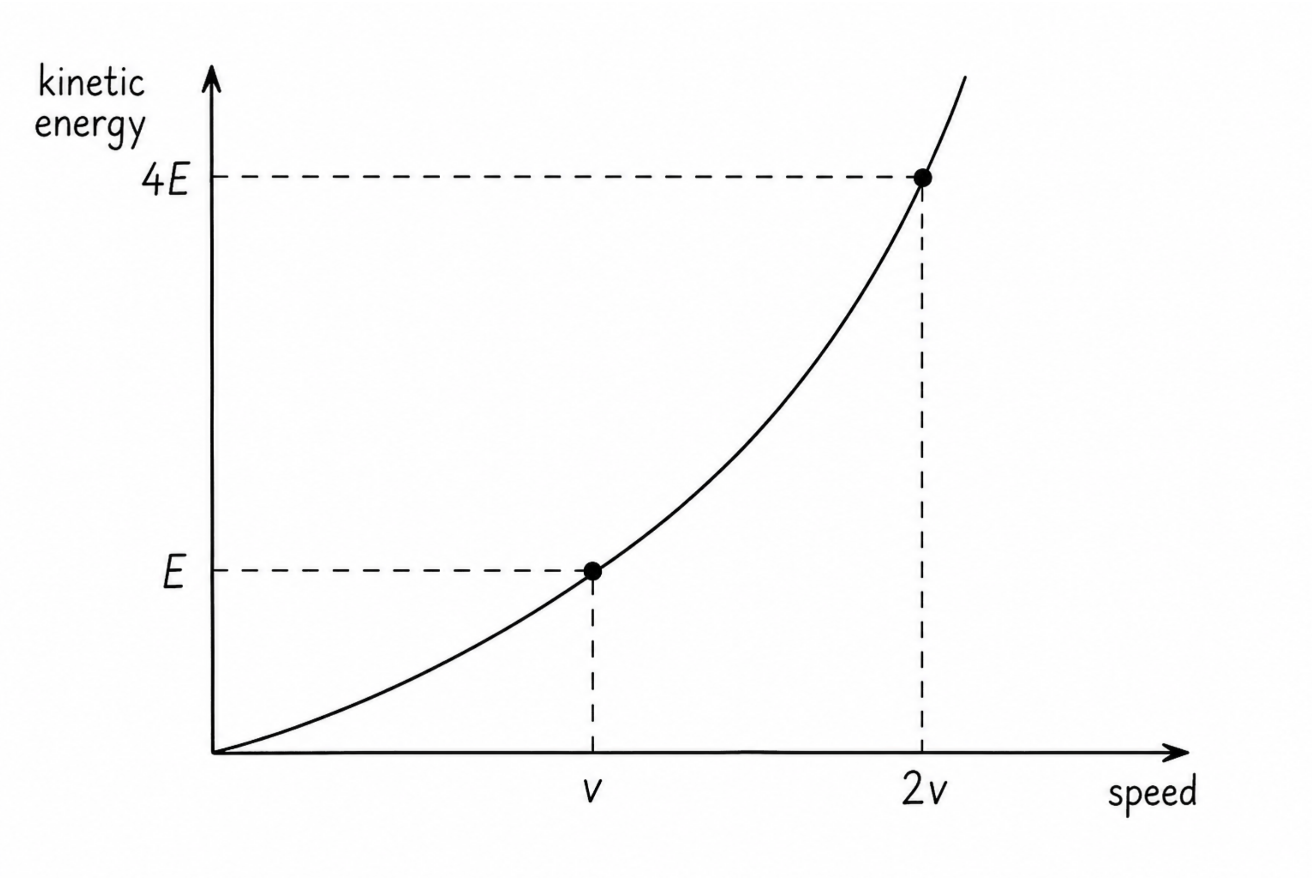
Kinetic energy is the energy an object has because it is moving. The formula $KE = \frac{1}{2}mv^2$ is one of the first equations students learn in physics, and it captures a deceptively important result: kinetic energy scales linearly with mass but with the SQUARE of velocity. Double the mass, double the kinetic energy. Double the speed, quadruple the kinetic energy. This single quadratic relationship explains why car accidents at 60 mph are far worse than at 30 mph, why air resistance becomes brutal at highway speeds, and why kinetic-energy weapons in physics are so disproportionately powerful.



Kinetic energy KE = 1/2 m v^2, the energy a moving object carries by virtue of its motion.

The Equation

The kinetic energy of an object moving in translation is:



The square in one picture: doubling the speed quadruples the energy, and with it the braking distance.

$$KE = \frac{1}{2}mv^2$$

Where:

- KE = kinetic energy, in joules (J)
- m = mass of the object, in kilograms (kg)
- v = speed of the object, in meters per second (m/s)

One joule is a small amount of energy on a human scale, roughly the energy of dropping a small apple about 10 cm. A 70 kg person running at 5 m/s has kinetic energy $\frac{1}{2} \times 70 \times 25 = 875$ J. A 1,500 kg car at 30 m/s (about 108 km/h) has $\frac{1}{2} \times 1500 \times 900 = 675{,}000$ J, nearly a thousand times more.

Why the Velocity-Squared Term Matters

The quadratic in velocity is the single most consequential feature of the kinetic energy equation. It means doubling the speed quadruples the energy. Tripling the speed multiplies energy by nine.

Practical consequences:

- **Car safety.** A 60 mph collision delivers 4× the energy of a 30 mph collision into the same car body. Braking distance scales with the square of speed, going twice as fast needs four times the stopping distance.
- **Bullets.** A heavy slow bullet and a light fast bullet can have similar momentum but very different kinetic energies. Wounding power tracks kinetic energy more closely than momentum.
- **Energy storage.** Flywheels store kinetic energy in proportion to the square of angular velocity. Doubling the rpm quadruples the stored energy.
- **Wind power.** A wind turbine's power output scales with v^3 (because energy scales with v^2 and the air-mass-per-second through the blades also scales with v). 20% more wind speed = 73% more power.

Deriving the Equation from First Principles

The kinetic energy formula falls out of the work-energy theorem. Work done by a constant force F over a distance d is $W = Fd$. For an object accelerating from rest under that force, Newton's second law gives $F = ma$, and the kinematic equation $v^2 = 2ad$ (starting from $v_0 = 0$) gives $d = v^2/(2a)$.

Substituting:

$$W = Fd = (ma) \cdot \frac{v^2}{2a} = \frac{1}{2}mv^2$$

The work done equals the kinetic energy gained. So $KE = \frac{1}{2}mv^2$ is just a definition that makes the work-energy theorem clean.

Kinetic Energy is Frame-Dependent

Unlike mass, kinetic energy depends on the frame of reference of the observer. A passenger sitting still in a moving train has zero kinetic energy in the train's reference frame but a substantial amount in the ground's reference frame. There is no 'true' kinetic energy of an object in absolute terms, it depends on who's watching and from where.

This is why kinetic-energy calculations always specify the reference frame implicitly (usually the ground frame for everyday problems). When two objects collide, you can analyze the collision in any inertial frame, the math is easiest in the center-of-mass frame, where total momentum is zero.

Rotational Kinetic Energy

Spinning objects also have kinetic energy, distinct from translational motion. For a rigid body rotating about a fixed axis:

$$KE_{rot} = \frac{1}{2}I\omega^2$$

Where I is the moment of inertia (the rotational analog of mass) and ω is the angular velocity in radians per second. Notice the same quadratic structure as translational KE. A rolling ball has both, translational KE for its center-of-mass motion and rotational KE for its spin.

Conservation of Kinetic Energy

Kinetic energy is conserved only in elastic collisions, collisions where no energy is converted to heat, deformation, or sound. Perfect elastic collisions are rare in macroscopic physics (close approximations: two hard steel balls, billiard ball-on-ball at low speed).

Most real collisions are inelastic, kinetic energy decreases as some converts to other forms. A perfectly inelastic collision (objects stick together) loses the maximum possible kinetic energy consistent with conservation of momentum.

Example. A 2 kg ball at 10 m/s hits a stationary 3 kg ball and they stick together. Momentum is conserved: $2 \times 10 + 3 \times 0 = 5v_f$, so $v_f = 4$ m/s. Initial KE: $\frac{1}{2} \times 2 \times 100 = 100$ J. Final KE: $\frac{1}{2} \times 5 \times 16 = 40$ J. The 60 J difference became heat and deformation.

Relativistic Kinetic Energy

At speeds approaching the speed of light, the simple $\frac{1}{2}mv^2$ formula breaks down. Einstein's special relativity gives:

$$KE = (\gamma - 1)mc^2 \quad \text{where} \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

For $v \ll c$, this reduces back to the classical $\frac{1}{2}mv^2$ (verify by Taylor-expanding γ). For $v \to c, \gamma \to \infty$, meaning infinite kinetic energy is required to reach the speed of light. This is the relativistic reason no massive object can ever reach c.

Related study notes: Newton's Laws of Motion, Special Relativity, Simple Harmonic Motion, Laws of Thermodynamics.

Practice Questions

Work each question before reading its solution. The set runs from direct recall and substitution to the applied questions that exams actually use to separate grades.

Q1. Compute the kinetic energy of a 1,200 kg car at 20 m/s (72 km/h).

SOLUTION

$KE = \frac{1}{2}mv^2 = 0.5 \times 1200 \times 400 = 240{,}000\text{ J} = 240\text{ kJ}$. About the energy in a 57-calorie snack, delivered in one instant if the car hits a wall.

Q2. The same car doubles its speed to 40 m/s. What happens to its kinetic energy, and why does this matter on the road?

SOLUTION

Energy quadruples to 960 kJ, because v enters squared. Braking distance scales with energy, so doubling speed roughly quadruples the distance needed to stop. The square is the physics behind speed limits.

Q3. A 0.145 kg baseball at 40 m/s and a 7,000 kg truck at 1 m/s: which carries more kinetic energy?

SOLUTION

Ball: $0.5 \times 0.145 \times 1600 = 116$ J. Truck: $0.5 \times 7000 \times 1 = 3500$ J. The crawling truck wins by 30 times, but note the ball beats it per kilogram a thousandfold: mass enters linearly, speed squared.

Q4. State the work-energy theorem and use it: a net 500 N force pushes a 50 kg crate from rest through 8 m. Find its final speed.

SOLUTION

Net work equals the change in kinetic energy: $W = \Delta KE$. Work $= 500 \times 8 = 4000\text{ J} = \frac{1}{2}(50)v^2$, so $v^2 = 160$ and $v \approx 12.6$ m/s. No kinematics equations needed; energy bookkeeping replaces them.

Q5. From what height must you drop a car to match its kinetic energy at 100 km/h (27.8 m/s)?

SOLUTION

Set $mgh = \frac{1}{2}mv^2$: $h = \frac{v^2}{2g} = \frac{772}{19.6} \approx 39$ m, a 13-story building. Mass cancels; the answer is the same for any object. Highway speed is a 13-floor fall in disguise.

Q6. A 9 g bullet leaves a rifle at 800 m/s. Compute its kinetic energy and compare it with the 240 kJ car of Question 1.

SOLUTION

$KE = 0.5 \times 0.009 \times 640{,}000 = 2880$ J. The car carries 80 times more energy, yet the bullet is deadlier: it delivers its 2.9 kJ into square millimeters, while the car spreads 240 kJ over crumple zones. Energy density, not just energy, decides damage.

Q7. Kinetic energy can never be negative, but a change in kinetic energy can. Explain both statements.

SOLUTION

$KE = \frac{1}{2}mv^2$ squares the speed, so it is zero or positive regardless of direction; energy has no direction to be negative in. But ΔKE is negative whenever something slows, meaning the net work on it was negative: friction and brakes do negative work, draining speed into heat.

Q8. Two identical cars collide head-on at 50 km/h each. Is the crash energy the same as hitting a rigid wall at 100 km/h?

SOLUTION

No. Each car carries the KE of 50 km/h, so the pair dissipates $2 \times KE_{50}$. The 100 km/h wall crash dissipates $KE_{100} = 4 \times KE_{50}$ in 1 car: twice the total, all in your crumple zone. By symmetry, the head-on equals hitting a wall at 50, not 100. The squared law breaks the intuition.

Q9. A pendulum swings from a 30 cm height. Ignoring friction, how fast is the bob at the lowest point?

SOLUTION

All potential energy converts: $v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.3} = \sqrt{5.88} \approx 2.4$ m/s. Conversion between mgh and $\frac{1}{2}mv^2$, in both directions, is the workhorse of mechanics problems.

Q10. At what speed does a 70 kg sprinter match the kinetic energy of a 70 g arrow flying at 60 m/s?

SOLUTION

Arrow: $0.5 \times 0.07 \times 3600 = 126$ J. Sprinter: $0.5 \times 70 \times v^2 = 126$ gives $v^2 = 3.6$, $v = 1.9$ m/s, a jog. A thousand times the mass needs only $\sqrt{1000} \approx 32$ times less speed for equal energy: the square root soft-pedals mass differences.

Answer Key

QUICK ANSWERS

Q1 240 kJ **Q2** ×4: 960 kJ **Q3** truck: 3,500 J vs 116 J **Q4** $v = \sqrt{160} \approx 12.6$ m/s **Q5** about 39 m **Q6** 2,880 J, about 1/80 of the car **Q7** $v^2 \geq 0$; slowing means negative net work **Q8** no: head-on = wall at 50 km/h each **Q9** $\approx$ 2.4 m/s **Q10** about 1.9 m/s

Revision Sheet

The formula

$KE = \frac{1}{2}mv^2$. Joules; 1 J = 1 kg·m^2/s^2.

Energy exchange

$mgh \leftrightarrow \frac{1}{2}mv^2$: falling from h gives $v = \sqrt{2gh}$, mass cancels.

The square

Double the speed: 4 times the energy and roughly 4 times the braking distance. Triple: 9 times.

Scales

Baseball pitch ~120 J; bullet ~3 kJ; car at 72 km/h ~240 kJ.

Work-energy theorem

$W_{net} = \Delta KE$. Positive net work speeds up; negative slows down.

Not a vector

KE has magnitude only and is never negative; only its change can be.

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