SEQUENCES

Geometric Progression

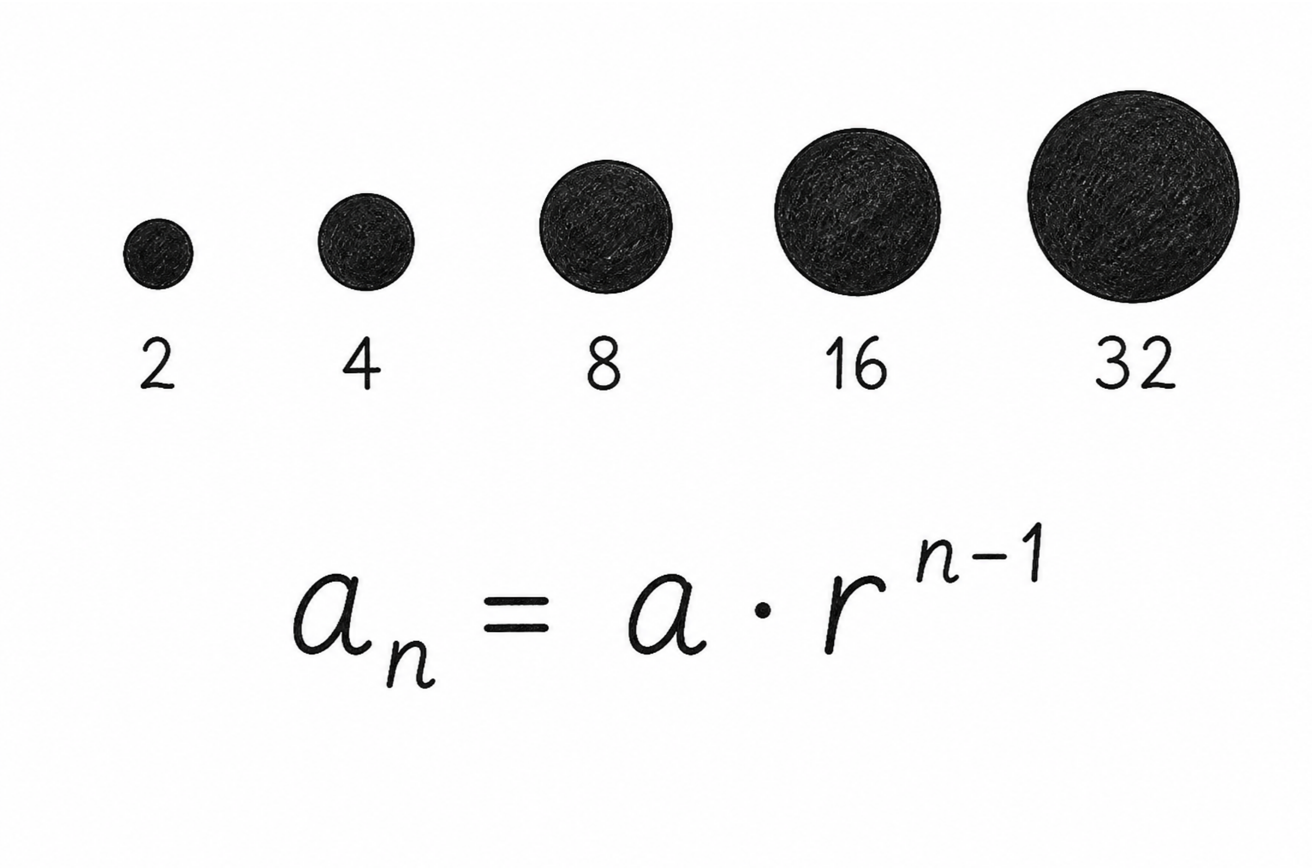
Constant-ratio sequences, the sum formulas including infinite series, and where compounding shows up, with a 10-question practice set and full solutions.

SUBJECT	LEVEL	INCLUDES
Mathematics	High school and early college	Practice set, answer key, revision sheet

Read this note online, with updates: gauravtiwari.org/study-notes/geometric-progression

INSIDE

A geometric progression (GP) is a sequence in which each term is obtained by multiplying the previous one by a fixed non-zero number called the common ratio r. Unlike an arithmetic progression, which grows by repeated addition, a GP grows (or shrinks) by repeated multiplication. This makes GPs the natural language of compound interest, population growth, radioactive decay, and any process where the change at each step is proportional to the current value.



Geometric progression: each term equals the previous one multiplied by a common ratio r.

Definition and General Term

A geometric progression has the form: $a, ar, ar^2, ar^3, \ldots$ where a is the first term and r is the common ratio. The n-th term is:

$$a_n = a \cdot r^{n-1}$$

You can identify a GP by checking that the ratio between consecutive terms is constant: $\frac{a_2}{a_1} = \frac{a_3}{a_2} = \cdots = r$.

Worked Examples

- **2, 4, 8, 16, 32, ...:** $a = 2, r = 2$. The 10th term is $2 \cdot 2^9 = 1024$.
- **81, 27, 9, 3, 1, $\frac{1}{3}$, ...:** $a = 81, r = \frac{1}{3}$. Shrinking GP.
- **5, -10, 20, -40, ...:** $a = 5, r = -2$. Alternating signs.
- **1000, 1050, 1102.5, ...:** $a = 1000, r = 1.05$. Compound interest at 5%.

Sum of a Finite GP

The sum of the first n terms of a GP with $r \neq 1$ is:

$$S_n = a \cdot \frac{1 - r^n}{1 - r}$$

Example: sum of $2 + 4 + 8 + \cdots + 1024$ (10 terms with $a = 2, r = 2$) is $2 \cdot \frac{1-2^{10}}{1-2} = 2 \cdot \frac{-1023}{-1} = 2046$.

Sum of an Infinite GP

If $|r| < 1$, the terms shrink toward zero and the infinite sum converges:

$$S_\infty = \frac{a}{1 - r} \quad \text{for } |r| < 1$$

Example: $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots = \frac{1}{1-1/2} = 2$. This is Zeno's paradox resolved: an infinite number of shrinking steps adds to a finite total.

If $|r| \geq 1$, the infinite series diverges (the terms don't shrink) and no finite sum exists.

Geometric Mean

In a GP, every term is the geometric mean of its neighbors: $a_n = \sqrt{a_{n-1} \cdot a_{n+1}}$. The geometric mean of two positive numbers x and y is $\sqrt{xy}$, the side length of a square with the same area as a rectangle of sides x and y.

Real-World Applications

- **Compound interest.** Money invested at rate r per period grows as $P, P(1+r), P(1+r)^2, \ldots$, a GP with ratio $1+r$.
- **Radioactive decay.** The amount remaining after each half-life is half the previous amount, a shrinking GP with $r = \frac{1}{2}$.
- **Population growth.** Idealized populations grow geometrically when resources are unlimited.
- **Computer science.** Binary trees, algorithm complexity (doubling work each step), and exponential time complexity all rest on GP behavior.
- **Music.** Equal-tempered scales use a GP of frequency ratios with $r = 2^{1/12} \approx 1.0595$, twelve semitones double the frequency.

Related study notes: Fibonacci Sequence, Exponential Function, Logarithms, Compound Interest.

Practice Questions

Work each question before reading its solution. The set runs from direct recall and substitution to the applied questions that exams actually use to separate grades.

Q1. For the GP 3, 6, 12, 24, ..., identify a and r, and find the 10th term.

SOLUTION

$a = 3$, common ratio $r = 2$. $a_{10} = ar^9 = 3 \times 512 = 1536$. Unlike an AP where you add d repeatedly, in a GP you multiply by r repeatedly, which is exactly why GPs explode (or collapse) so much faster than APs.

Q2. A GP has $a_3 = 20$ and $a_6 = 160$. Find r and a.

SOLUTION

$\frac{a_6}{a_3} = r^3 = \frac{160}{20} = 8$, so $r = 2$. Then $a_3 = ar^2 = 4a = 20$, giving $a = 5$. Dividing 2 terms cancels a and isolates a power of r, the standard opening move for GP problems.

Q3. Sum the first 8 terms of the GP 2, 6, 18, ...

SOLUTION

$S_n = a\frac{r^n - 1}{r - 1} = 2 \times \frac{3^8 - 1}{3 - 1} = 2 \times \frac{6560}{2} = 6560$. The finite-sum formula handles any ratio except $r = 1$, where the sum is trivially na instead.

Q4. Find the sum of the infinite GP $4 + 2 + 1 + 0.5 + \cdots$, and explain why this sum does NOT diverge despite having infinitely many terms.

SOLUTION

$|r| = 0.5 < 1$, so $S_\infty = \frac{a}{1 - r} = \frac{4}{0.5} = 8$. Each term shrinks fast enough that the total added by all remaining terms after any point becomes negligible; the partial sums converge to a finite limit even though the series never technically stops. This only works when $|r| < 1$.

Q5. Convert the repeating decimal $0.\overline{3}$ (0.3333...) to a fraction using the infinite GP sum formula.

SOLUTION

$0.3333\ldots = 0.3 + 0.03 + 0.003 + \cdots$, a GP with $a = 0.3$, $r = 0.1$. $S_\infty = \frac{0.3}{1 - 0.1} = \frac{0.3}{0.9} = \frac{1}{3}$. Every repeating decimal is secretly an infinite geometric series, which is exactly why every repeating decimal equals some fraction.

Q6. \{}$1,000 is invested at 8% annual interest, compounded annually. Write the balance after n years as a GP term, and find the balance after 5 years.

SOLUTION

Balance follows $A_n = 1000(1.08)^n$, a GP with $a = 1000$, $r = 1.08$ (n starting from year 0). After 5 years: $1000(1.08)^5 \approx 1000 \times 1.4693 \approx 1{,}469.33$. Compound interest is literally a geometric progression: each year multiplies the balance by the same growth factor.

Q7. A ball is dropped from 10 m and bounces back to 60% of its previous height each time. Find the total vertical distance it travels before effectively coming to rest.

SOLUTION

Falls: 10 (first drop) then each bounce goes UP and back DOWN, so each bounce after the first contributes 2× its rise height. Total $= 10 + 2(6) + 2(3.6) + 2(2.16) + \cdots = 10 + 2 \times \dfrac{6}{1 - 0.6} = 10 + 2(15) = 40$ m. The infinite-sum formula handles the "bounces forever, but converges" physical reality cleanly.

Q8. Insert 2 geometric means between 4 and 108 (i.e., find x, y so that 4, x, y, 108 forms a GP).

SOLUTION

Four terms means $r^3 = 108/4 = 27$, so $r = 3$. Terms: $x = 4 \times 3 = 12$, $y = 12 \times 3 = 36$. Check: $4, 12, 36, 108$ with ratio 3 throughout. Inserting k geometric means always uses r^{k+1} = (last term)/(first term).

Q9. Why does compound interest ("the eighth wonder of the world," reportedly per popular finance lore) so drastically outperform simple interest over long periods? Compare \{}$1,000 at 10% simple vs compound interest over 30 years.

SOLUTION

Simple interest: $1000 + 1000(0.10)(30) = 1000 + 3000 = 4000$, linear growth (an AP in disguise). Compound: $1000(1.10)^{30} \approx 17{,}449$, a GP, over 4 times larger. Compounding lets interest itself earn interest each period, a multiplicative ratchet that simple interest's purely additive structure can never replicate, and the gap widens explosively with time.

Q10. A chessboard has 64 squares. If 1 grain of rice is placed on square 1, 2 on square 2, 4 on square 3, doubling each time, find the total grains on the entire board, and give a sense of the scale.

SOLUTION

$S_{64} = \dfrac{1(2^{64}-1)}{2-1} = 2^{64}-1 \approx 1.8 \times 10^{19}$ grains. At roughly 25 mg per grain, that is about 4.6×10^{14} kg of rice, vastly more than all the rice ever harvested in human history. The classic "doubling on a chessboard" legend is the canonical illustration of how deceptively fast geometric growth compounds.

Answer Key

QUICK ANSWERS

Q1 $a_{10} = 1536$ **Q2** $a = 5, r = 2$ **Q3** 6560 **Q4** $S_\infty = 8$ **Q5** 1/3

Q6 $\approx$ \$1,469.33 **Q7** 40 m **Q8** $x = 12, y = 36$ **Q9** simple: \{}\$4,000; compound: ≈\{}\$17,449

Q10 $2^{64} - 1 \approx 1.8 \times 10^{19}$ grains

Revision Sheet

nth term

$a_n = ar^{n-1}$: multiply by r each step, not add.

Finite sum

$S_n = a\dfrac{r^n - 1}{r - 1}$ (or na if $r = 1$).

Infinite sum

$S_\infty = \dfrac{a}{1 - r}$, valid ONLY when $|r| < 1$.

Repeating decimals

Every repeating decimal is an infinite GP; convert via S_∞.

Compound interest

$A = P(1 + i)^n$ is a GP: multiplicative growth beats simple (additive) interest over time.

Inserting means

k geometric means between a and b: $r^{k+1} = b/a$.

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