ELECTROMAGNETISM

Gauss's Law: Electric Flux, Symmetry, and Examples

Electric flux through a closed surface, why symmetry makes fields easy, and Gauss's law's 4 classic geometries, with a 10-question practice set and full solutions.

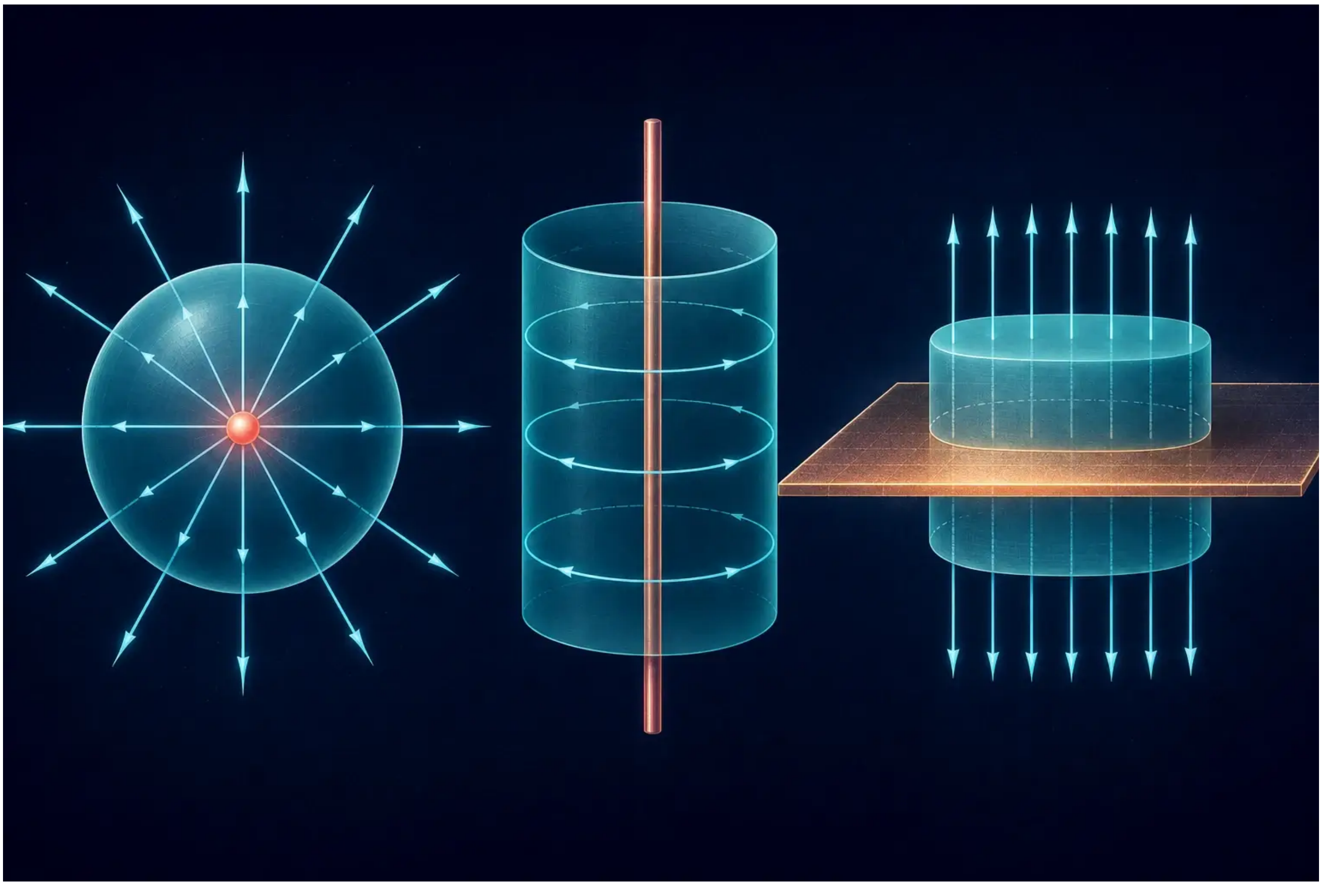
SUBJECT	LEVEL	INCLUDES
Physics	High school and early college	Practice set, answer key, revision sheet

Read this note online, with updates: gauravtiwari.org/study-notes/gauss-law

INSIDE

Gauss's law states that the net electric flux through any closed surface equals the enclosed charge divided by the vacuum permittivity. The law is always true in electrostatics, but it is only a shortcut for finding the field when symmetry controls the field's magnitude and direction.

A Gaussian surface is imaginary. It does not block the field, and it does not have to match a physical boundary. You choose it so the flux integral becomes simple.



A Gaussian surface is useful when the charge distribution makes the electric field constant or perpendicular in the right places.

What Is Gauss's Law?

$$\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\varepsilon_0}$$

The integral is over a closed surface S. The area vector $d\mathbf{A}$ points outward. Q_{enc} includes the net charge inside the surface, with signs. Charges outside can affect $\mathbf{E}$ on the surface, but their net contribution to the closed-surface flux is zero.

QUANTITY	MEANING	SI UNIT
$\mathbf{E}$	Electric field	N/C or V/m
$d\mathbf{A}$	Outward vector area element	m^2
Φ_E	Electric flux	$N\,m^2/C$
Q_{enc}	Net enclosed charge	C
ε_0	Vacuum permittivity	F/m

The OpenStax guide to applying Gauss's law shows the same three standard symmetry classes: spherical, cylindrical, and planar.

Electric Flux Before Gauss's Law

Flux measures how strongly a vector field passes through a surface. For a uniform field and a flat area

$$\Phi_E = \mathbf{E} \cdot \mathbf{A} = EA\cos\theta$$

Here θ is the angle between the electric field and the area normal, not the surface itself. A field perpendicular to the surface has maximum flux. A field tangent to the surface has zero flux.

- **Outward crossing:** positive flux.
- **Inward crossing:** negative flux.
- **Tangent field:** zero local flux because $\mathbf{E} \cdot d\mathbf{A} = 0$.
- **Closed surface:** only the net outward flux enters Gauss's law.

When Gauss's Law Actually Solves the Field

Gauss's law is useful for finding E only when symmetry lets you take E outside part of the integral or makes other parts contribute zero. The shape of the Gaussian surface alone creates no symmetry. The charge distribution must have the symmetry.

1. Identify whether the source has spherical, cylindrical, or planar symmetry.
2. Infer the allowed direction of the electric field.
3. Choose a closed surface on which E is constant where flux is nonzero.
4. Compute the flux and enclosed charge separately.
5. Apply Gauss's law and solve for E.
6. Check the direction, units, and limiting behaviour.

Spherical Symmetry: Point Charge and Charged Sphere

For a point charge Q at the centre of a spherical Gaussian surface of radius r, symmetry makes the field radial and constant in magnitude on the sphere.

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

This is the point-charge field from Coulomb's law. Outside any spherically symmetric charge distribution, the field has the same form with Q equal to the total enclosed charge.

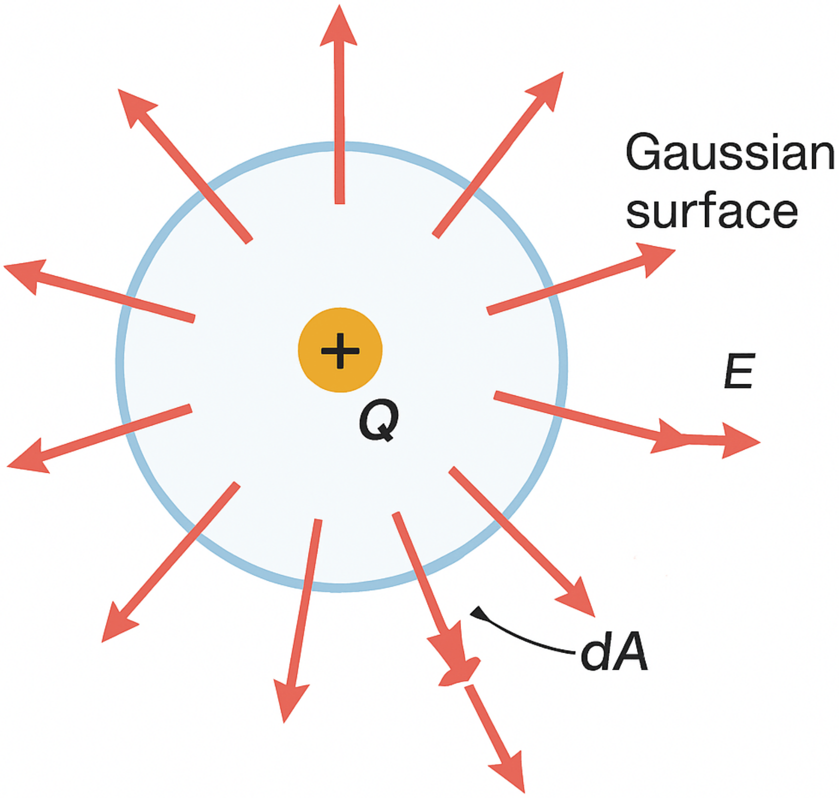
Uniformly Charged Solid Sphere

For a sphere of radius R and uniform volume charge density ρ, a Gaussian sphere with $r < R$ encloses $Q_{\text{enc}} = \rho(4\pi r^3/3)$. Therefore

$$E(r) = \frac{\rho r}{3\varepsilon_0} = \frac{Qr}{4\pi\varepsilon_0 R^3} \qquad (r < R)$$

Inside, the field rises linearly from zero at the centre. Outside, $E = Q/(4\pi\varepsilon_0 r^2)$. The two expressions agree at $r = R$.

Cylindrical Symmetry: Infinite Line Charge



Gauss's law: the total electric flux through a closed surface equals the enclosed charge divided by ε_0.

For an infinitely long line with linear charge density λ, choose a coaxial cylinder of radius r and length L. The field is radial. Flux through the end caps is zero because the field is tangent there.

$$E(2\pi rL) = \frac{\lambda L}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

The field falls as $1/r$, not $1/r^2$, because cylindrical area grows linearly with radius. A finite wire approaches this result only far from its ends and at distances small compared with its length.

Planar Symmetry: Infinite Sheet of Charge

For an infinite sheet with surface charge density σ, use a pillbox crossing the sheet. The field is perpendicular to the plane and has equal magnitude on both sides. Only the two flat faces contribute.

$$2EA = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

The ideal infinite-sheet field does not depend on distance. A large finite sheet behaves approximately this way near its centre when the observation distance is much smaller than the sheet dimensions.

Conductors in Electrostatic Equilibrium

Free charges in a conductor move until the internal electric field is zero. A Gaussian surface entirely within the conducting material therefore has zero flux and encloses zero net charge. Any excess charge resides on the surface.

- The electric field inside the conducting material is zero.
- The conductor is an equipotential.
- Excess charge lies on the surface.
- The field immediately outside is normal to the surface.
- In vacuum just outside, $E_\perp = \sigma/\varepsilon_0$ for the local surface charge density.

A hollow cavity inside a conductor is field-free only under the appropriate electrostatic conditions, such as no charge inside the cavity. Put a charge in the cavity and induced surface charges appear.

Why Outside Charges Give Zero Net Flux

A field line from an external charge that enters a closed surface must leave it again. The inward contribution is negative and the outward contribution is positive. Their total is zero even though the external charge can make the field large and nonuniform at individual points.

This is why zero enclosed charge does not imply zero electric field. It implies zero net flux. A uniform field passing through an empty box is the simplest example: as much flux enters as leaves.

Common Mistakes

- **Using total charge instead of enclosed charge:** only charge inside the chosen closed surface appears on the right side.
- **Assuming zero enclosed charge means zero field:** it means zero net flux.
- **Choosing a convenient shape without source symmetry:** the integral will not simplify just because the surface is a sphere.
- **Forgetting the end caps:** show why their dot product is zero or include their flux.
- **Using open surfaces:** Gauss's law uses a closed surface.
- **Ignoring sign:** enclosed negative charge produces negative net outward flux.

Related Physics Guides

Use these next if you want to connect this result with the surrounding physics:

- Coulomb's law
- The four fundamental forces
- Symmetry in physical laws

Key Takeaways

- Gauss's law is $\oint \mathbf{E} \cdot d\mathbf{A} = Q_{\text{enc}}/\varepsilon_0$.
- The law is universal, but solving for E requires strong symmetry.
- External charges can change the local field but add zero net closed-surface flux.
- Spherical, cylindrical, and planar symmetry give the standard useful Gaussian surfaces.
- Zero flux and zero field are different statements.

The quickest test is blunt: if you cannot state why E is constant on the chosen surface, Gauss's law probably will not save algebra.

Practice Questions

Work each question before reading its solution. The set runs from direct recall and substitution to the applied questions that exams actually use to separate grades.

Q1. State Gauss's law and explain what Φ_E and Q_{enc} represent.

SOLUTION

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

Φ_E is the total electric flux through a closed surface, informally how much field "flows out" through it. Q_{enc} is only the charge ENCLOSED by that surface; anything outside contributes zero net flux, even though it still bends the field locally.

Q2. Why does charge OUTSIDE a closed surface contribute zero net flux through it, even though its field lines do cross the surface?

SOLUTION

Every field line from an external charge that enters the surface on 1 side must exit somewhere else, since field lines from a charge outside cannot terminate inside a region containing no charge. Entering and exiting flux exactly cancel in the net sum, which is why only enclosed charge survives in the final total.

Q3. A point charge of 5 μC sits at the center of an imaginary sphere of radius 0.2 m. Find the total electric flux through the sphere.

SOLUTION

$\Phi_E = Q_{enc}/\varepsilon_0 = (5 \times 10^{-6})/(8.85 \times 10^{-12}) \approx 5.65 \times 10^5$ N·m^2/C. Notice the RADIUS never entered the calculation: Gauss's law depends only on enclosed charge, a genuinely surprising and powerful simplification.

Q4. Use Gauss's law to derive the electric field magnitude at distance r from a point charge Q, and show it reproduces Coulomb's law.

SOLUTION

Choose a spherical Gaussian surface of radius r. By symmetry E is uniform and radial, so $\Phi_E = E \times 4\pi r^2 =$

Q/ε_0, giving $E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{kQ}{r^2}$, exactly Coulomb's law. Gauss's law is not a separate physical fact from Coulomb's law; it is the same physics viewed through flux, and it derives Coulomb's law as a special case.

Q5. For a uniformly charged infinite line with linear charge density λ, use a cylindrical Gaussian surface to find $E(r)$.

SOLUTION

A coaxial cylinder of radius r, length L: flux exits only through the curved side (the flat ends have field parallel to them, contributing zero). $E \times 2\pi rL = \lambda L/\varepsilon_0$, so $E = \frac{\lambda}{2\pi\varepsilon_0 r}$. The field falls off as $1/r$, not $1/r^2$, because the "source" is spread along an infinite line rather than concentrated at a point.

Q6. For an infinite charged plane with surface charge density σ, use a pillbox Gaussian surface to find E, and explain why it does NOT depend on distance from the plane.

SOLUTION

A pillbox straddling the plane, flat faces of area A parallel to it: flux exits both faces equally, $2EA = \sigma A/\varepsilon_0$, giving $E = \frac{\sigma}{2\varepsilon_0}$, a CONSTANT. Field lines from an infinite plane never spread apart or converge (there is no edge to diverge toward), so intensity never dilutes with distance, unlike the point-charge or line-charge cases.

Q7. Why does the electric field inside a solid conductor in electrostatic equilibrium equal exactly zero, using Gauss's law reasoning?

SOLUTION

Choose a Gaussian surface just inside the conductor's surface. Free charges in a conductor redistribute until the field inside vanishes (otherwise charges would still be accelerating, contradicting equilibrium); by Gauss's law, zero field through that interior surface means zero enclosed charge, so all excess charge must reside on the outer SURFACE, none in the bulk. This is why a hollow metal box shields its interior from external fields (a Faraday cage).

Q8. A charged spherical shell (radius R, total charge Q) has field $E = 0$ everywhere inside it. Use Gauss's law to prove this without detailed integration.

SOLUTION

For any Gaussian sphere of radius $r < R$ drawn inside the shell, $Q_{enc} = 0$ (all the actual charge sits on the shell at radius R, outside this smaller sphere). By Gauss's law, $\Phi_E = 0/\varepsilon_0 = 0$, and by the shell's spherical symmetry E must be uniform over the Gaussian sphere, so $E = 0$ throughout the interior. This 1-line argument replaces pages of direct integration.

Q9. A solid sphere of radius R carries uniform charge Q throughout its volume. Find E at a point INSIDE the sphere, at distance $r < R$ from the center.

SOLUTION

Only the charge within radius r counts as enclosed: $Q_{enc} = Q\frac{r^3}{R^3}$ (charge scales with enclosed volume for uniform density). Then $E \times 4\pi r^2 = \frac{Qr^3/R^3}{\varepsilon_0}$, giving $E = \frac{kQr}{R^3}$: the field grows LINEARLY with r inside a uniformly charged sphere, in contrast to falling off as $1/r^2$ outside it.

Q10. Explain why Gauss's law, while always TRUE, is only USEFUL for calculating E directly in a handful of highly symmetric charge configurations (spherical, cylindrical, planar). What must you do for an arbitrary charge distribution?

SOLUTION

Pulling E out of the flux integral as a constant requires the Gaussian surface to be chosen where E is known in advance to be uniform in magnitude and either parallel or perpendicular to the surface everywhere, which only symmetric charge distributions guarantee. For an irregular distribution, no such convenient surface exists, and the flux integral cannot be simplified; the practical fallback is Coulomb's law integrated directly over the charge distribution, or numerical computation.

Answer Key

QUICK ANSWERS

Q1 flux out = enclosed charge / ε_0 **Q2** field lines that enter must also exit: net contribution cancels **Q3** $\approx 5.65 \times 10^5$ N·m²/C **Q4** $E = kQ/r^2$: recovers Coulomb's law **Q5** $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$ **Q6** $E = \dfrac{\sigma}{2\varepsilon_0}$: constant, independent of distance **Q7** zero interior field forces all excess charge to the surface **Q8** $Q_{enc} = 0$ inside $\Rightarrow$ $E = 0$ inside, by symmetry **Q9** $E = \dfrac{kQr}{R^3}$: grows linearly inside **Q10** symmetry alone lets E factor out of the integral; otherwise integrate Coulomb's law directly

Revision Sheet

The law

$\oint \vec{E} \cdot d\vec{A} = Q_{enc}/\varepsilon_0$: net flux depends only on ENCLOSED charge.

Conductors

Interior field is zero in equilibrium; all excess charge sits on the outer surface (Faraday cage).

The trick

Choose a Gaussian surface matching the charge symmetry so E is constant over it and pulls out of the integral.

Shell theorem

Zero field strictly inside a uniformly charged shell, by $Q_{enc} = 0$ plus symmetry.

4 classic results

Point: $E = kQ/r^2$. Line: $E = \lambda/2\pi\varepsilon_0 r$. Plane: $E = \sigma/2\varepsilon_0$ (constant!). Sphere interior: $E \propto r$.

Limitation

Only symmetric distributions let E factor out; irregular charges need direct Coulomb integration.

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