WAVES

Doppler Effect

Why pitch shifts with relative motion, the formula, and applications from sirens to cosmology, with a 10-question practice set and full solutions.

SUBJECT	LEVEL	INCLUDES
Physics	High school and early college	Practice set, answer key, revision sheet

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INSIDE

The Doppler effect is the apparent change in the frequency of a wave when there is relative motion between the source and the observer. You have heard it on every ambulance siren that has passed you, the pitch rises as the ambulance approaches and drops as it recedes. The same phenomenon shifts the colors of stars moving toward or away from Earth (redshift and blueshift), produces radar guns that measure car speeds, and lets weather forecasters track storm motion. Christian Doppler proposed the effect for sound waves in 1842; Hippolyte Fizeau extended it to light in 1848. The physics is the same for any kind of wave.

The Doppler effect, the observer in front of the source hears a higher pitch (compressed wavefronts), the observer behind hears a lower pitch (stretched wavefronts).

What's Happening Physically

Sound (or any wave) travels through a medium as a series of compressions and rarefactions. The frequency you perceive is determined by how often these compressions arrive at your ear. If the source is stationary, the wavefronts are evenly spaced concentric spheres expanding outward at the speed of sound.

Now make the source move. As it emits each new wavefront, the source has shifted slightly. Wavefronts emitted in the direction of motion become bunched closer together, the source 'catches up' to the wavefronts it just made. Wavefronts emitted behind the source become spread farther apart. The actual sound waves are unchanged at the source, but the spacing as observed from a stationary point depends on which direction you are watching from.

Closer-together wavefronts mean more arrive per second, higher observed frequency. Farther-apart wavefronts mean fewer per second, lower observed frequency. The effect grows with the source's speed.

The Equation for Sound

For sound waves moving through air (or any other medium), the observed frequency depends on whether the source and/or observer are moving and in which direction:

$$f_{obs} = f_{source} \cdot \frac{v \pm v_{obs}}{v \mp v_{source}}$$

Where:

- f_{obs} = observed frequency
- f_{source} = frequency emitted by source
- v = speed of sound in the medium (~343 m/s in air at room temperature)
- v_{obs} = speed of observer (positive if moving TOWARD source, negative if AWAY)
- v_{source} = speed of source (positive if moving TOWARD observer, negative if AWAY)

The sign convention is the confusing part, easier to remember the qualitative rule: motion that DECREASES the distance between source and observer increases the observed frequency; motion that INCREASES the distance decreases the frequency.

Worked Examples

Example 1. An ambulance siren emits sound at 700 Hz while moving toward a stationary observer at 30 m/s. What does the observer hear?

$f_{obs} = 700 \times 343/(343 - 30) = 700 \times 343/313 \approx 767$ Hz. The pitch rises by 9.5%.

Example 2. After the ambulance passes, it recedes at 30 m/s. Now the observer hears:

$f_{obs} = 700 \times 343/(343 + 30) = 700 \times 343/373 \approx 644$ Hz. The pitch drops by 8%.

The total pitch change as the ambulance passes is about 18%, quite audible. The classic 'wee-OO-wee-OO' rising-then-falling tone is the Doppler effect made hearable.

Doppler Effect for Light

Light waves obey a similar but relativistic version of the Doppler effect. For a source moving at velocity v (with positive v meaning approaching the observer), the observed wavelength is:

$$\frac{\lambda_{obs}}{\lambda_{source}} = \sqrt{\frac{1 - v/c}{1 + v/c}}$$

Where c is the speed of light. For $v \ll c$, this reduces to the simpler approximation $\Delta\lambda/\lambda \approx -v/c$.

Sources approaching the observer have shorter observed wavelengths, the light shifts toward the blue end of the spectrum (blueshift). Sources receding have longer observed wavelengths, the light shifts toward the red (redshift).

Cosmological Redshift

In the 1920s, Edwin Hubble showed that distant galaxies show redshifts proportional to their distance, farther galaxies are redshifted more. The relationship, Hubble's law, is $v = H_0 d$, where H_0 is the Hubble constant. The interpretation: the universe is expanding, and the further away a galaxy is, the faster it is moving away from us. This was the first direct evidence for the Big Bang.

Cosmological redshift is technically not a Doppler effect, it is caused by the expansion of space itself, not the galaxies' motion through space. But the math looks similar and the relationship between observed wavelength shift and recession velocity is qualitatively the same.

Modern Applications

- **Police radar guns.** Emit microwaves of known frequency at a moving car. The microwaves reflect off the car (which acts as a moving source from the gun's perspective) and the gun measures the Doppler-shifted return frequency, calculating the car's speed.
- **Weather Doppler radar.** Emits radio waves at a precipitation field; the Doppler shift of the returning signal reveals the velocity of rain or hail toward or away from the radar. Used to detect mesocyclones and tornadoes.
- **Medical ultrasound (Doppler echocardiography).** Measures blood flow velocity inside the heart and arteries by Doppler-shifting ultrasound off the moving red blood cells. Non-invasive and routinely used to diagnose heart conditions.
- **Exoplanet detection (radial velocity method).** A planet orbiting a star pulls the star slightly. The star's spectrum is Doppler-shifted periodically as it wobbles. By 2026, over 5,000 confirmed exoplanets have been detected this way.
- **GPS and satellite communications.** Satellites moving at orbital velocities Doppler-shift their signals. GPS receivers must compensate for this shift to compute accurate positions.

The Sonic Boom

When a source moves through a medium faster than the wave speed in that medium, something dramatic happens. Each wavefront stacks up at the location where the source emitted it, and the source itself races past. Behind the source, the wavefronts pile up into a shock cone, the Mach cone. When that cone passes an observer, they hear a single loud crack: the sonic boom.

The same principle in optics is called Cherenkov radiation. Charged particles moving through a transparent medium faster than the speed of light in that medium (which is slower than c) emit a cone of blue light. The eerie blue glow of nuclear reactor coolant pools is Cherenkov radiation.

Related study notes: Special Relativity, Maxwell's Equations, Black Hole, Exponential Function.

Practice Questions

Work each question before reading its solution. The set runs from direct recall and substitution to the applied questions that exams actually use to separate grades.

Q1. Explain the Doppler effect mechanism in words: why does an approaching source sound higher-pitched?

SOLUTION

As the source moves toward you, each successive wave crest is emitted from a position slightly closer than the last, compressing the crests together in the direction of travel. Shorter wavelength arriving at your ear, at the same wave speed, means more crests per second: higher frequency, higher pitch. Receding does the reverse, stretching wavelength and lowering pitch.

Q2. Write the Doppler formula for a moving source and stationary observer, and state the sign convention.

SOLUTION

$f_{obs} = f_{src}\dfrac{v}{v \mp v_s}$, where v is the wave speed, v_s the source speed. Minus in the denominator (source approaching) raises the observed frequency; plus (receding) lowers it. Getting the sign backward is the single most common Doppler mistake on exams.

Q3. An ambulance siren emits 700 Hz and approaches at 30 m/s. Find the frequency you hear (sound speed 340 m/s).

SOLUTION

$f_{obs} = 700 \times \dfrac{340}{340 - 30} = 700 \times \dfrac{340}{310} \approx 768$ Hz. The pitch rises about 10% while it approaches, a shift easily audible without instruments.

Q4. The same ambulance now recedes at 30 m/s. Find the frequency, and describe the total pitch swing as it passes you.

SOLUTION

$f_{obs} = 700 \times \dfrac{340}{370} \approx 643$ Hz. As the ambulance passes, pitch drops abruptly from about 768 Hz to about 643 Hz, a jump of roughly 125 Hz, which is exactly the classic ”eeeeyyoowww” siren-passing sound everyone recognizes.

Q5. Give the Doppler formula for a moving observer approaching a stationary source, and explain why it differs structurally from the moving-source case.

SOLUTION

$f_{obs} = f_{src}\dfrac{v+v_o}{v}$: the observer's own motion adds directly to the wave speed relative to them, changing the RATE of crest arrival, unlike the source case, which changes the wavelength ITSELF. Different physical mechanisms, similar-looking formulas: this asymmetry is exactly why sound Doppler has 2 distinct formulas while light Doppler (relativistic) collapses them into 1.

Q6. A weather radar station is stationary; a storm cell moves away at 20 m/s reflecting a 3 GHz signal. Why does Doppler weather radar measure this shift at all, given the "source" (the radar) never moves?

SOLUTION

The moving raindrops act as a moving reflector: they receive a Doppler-shifted signal as moving "observers," then re-emit it as moving "sources," so the round trip doubles the shift. Meteorologists invert this double Doppler shift to map wind velocity inside storms directly, which is how radar detects rotation inside a developing tornado before it touches down.

Q7. What is redshift in astronomy, and what did Edwin Hubble's 1929 measurements of galactic redshift establish?

SOLUTION

Light from a receding source stretches to longer (redder) wavelengths, the optical Doppler effect. Hubble found that nearly every galaxy's light was redshifted, and that the redshift grew with distance, meaning the universe is expanding and distant galaxies recede faster than nearby ones. This became Hubble's law and the founding observational evidence for the Big Bang model.

Q8. A star's spectral line, normally at 656.3 nm, is observed at 656.5 nm. Find the star's radial velocity (approximate, non-relativistic formula $v = c\,\Delta\lambda/\lambda$).

SOLUTION

$v = (3\times10^8)\times\dfrac{0.2}{656.3} \approx 91{,}400$ m/s ≈ 91 km/s, receding (redshifted, longer wavelength). Astronomers routinely measure stellar and galactic velocities this way, entirely from a wavelength shift on a spectrum, with no direct distance measurement involved.

Q9. What is a sonic boom, and how does it connect to the Doppler formula's behavior as source speed approaches wave speed?

SOLUTION

As $v_s \to v$ in the Doppler denominator $(v - v_s)$, the observed frequency diverges toward infinity: the source is catching up to its own emitted wavefronts, piling them into a single shockwave cone (the Mach cone) instead of separate crests. The boom is the sudden arrival of that entire compressed pressure wall at once, and it explains why the formula itself "breaks" exactly at the speed of sound.

Q10. Police radar measures a car's speed via Doppler shift on a reflected radio wave. If a car reflects a 10.5 GHz signal back with a shift of 1,400 Hz, estimate its speed (use $v = c\,\Delta f/(2f)$ for the round-trip reflection case).

SOLUTION

$v = \dfrac{(3\times10^8)\times1400}{2\times10.5\times10^9} = \dfrac{4.2\times10^{11}}{2.1\times10^{10}} \approx 20\,\mathrm{m/s} \approx 72\,\mathrm{km/h}$. The factor of 2 accounts for the double Doppler shift, exactly as in weather radar: the moving car first receives, then re-reflects the signal, shifting it twice.

Answer Key

QUICK ANSWERS

Q1 crests bunch together ahead of the moving source **Q2** $f_{obs} = f_{src}\frac{v}{v \mp v_s}$ **Q3** $\approx$ 768 Hz **Q4** $\approx$ 643 Hz; a ~125 Hz drop at passing **Q5** $f_{obs} = f_{src}\frac{v+v_o}{v}$; observer motion adds crest-arrival rate directly **Q6** raindrops shift the signal twice: as receiver, then as re-emitter **Q7** light stretches to red for receding sources; proved cosmic expansion **Q8** $\approx$ 91 km/s, receding **Q9** the Doppler formula diverges as $v_s \to v$: crests pile into a shockwave **Q10** $\approx$ 20 m/s $\approx$ 72 km/h

Revision Sheet

The mechanism

Motion compresses or stretches wave crests: approach raises frequency, recession lowers it.

Double shift

Radar (weather, police) reflects off a moving target: shift happens twice, round trip.

Moving source

$f_{obs} = f_{src}\dfrac{v}{v \mp v_s}$ (minus = approaching, plus = receding).

Light and cosmology

Redshift $v \approx c\,\Delta\lambda/\lambda$; Hubble's redshift-distance relation revealed cosmic expansion.

Moving observer

$f_{obs} = f_{src}\dfrac{v \pm v_o}{v}$: observer motion adds directly to crest-arrival rate.

The limit

As $v_s \to v$, frequency diverges: crests pile into a shockwave, the sonic boom.

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