ELECTROSTATICS

Coulomb's Law: Formula, Direction, and Examples

The force law, superposition, the comparison with gravity, the field picture, and a 10-question practice set with full solutions.

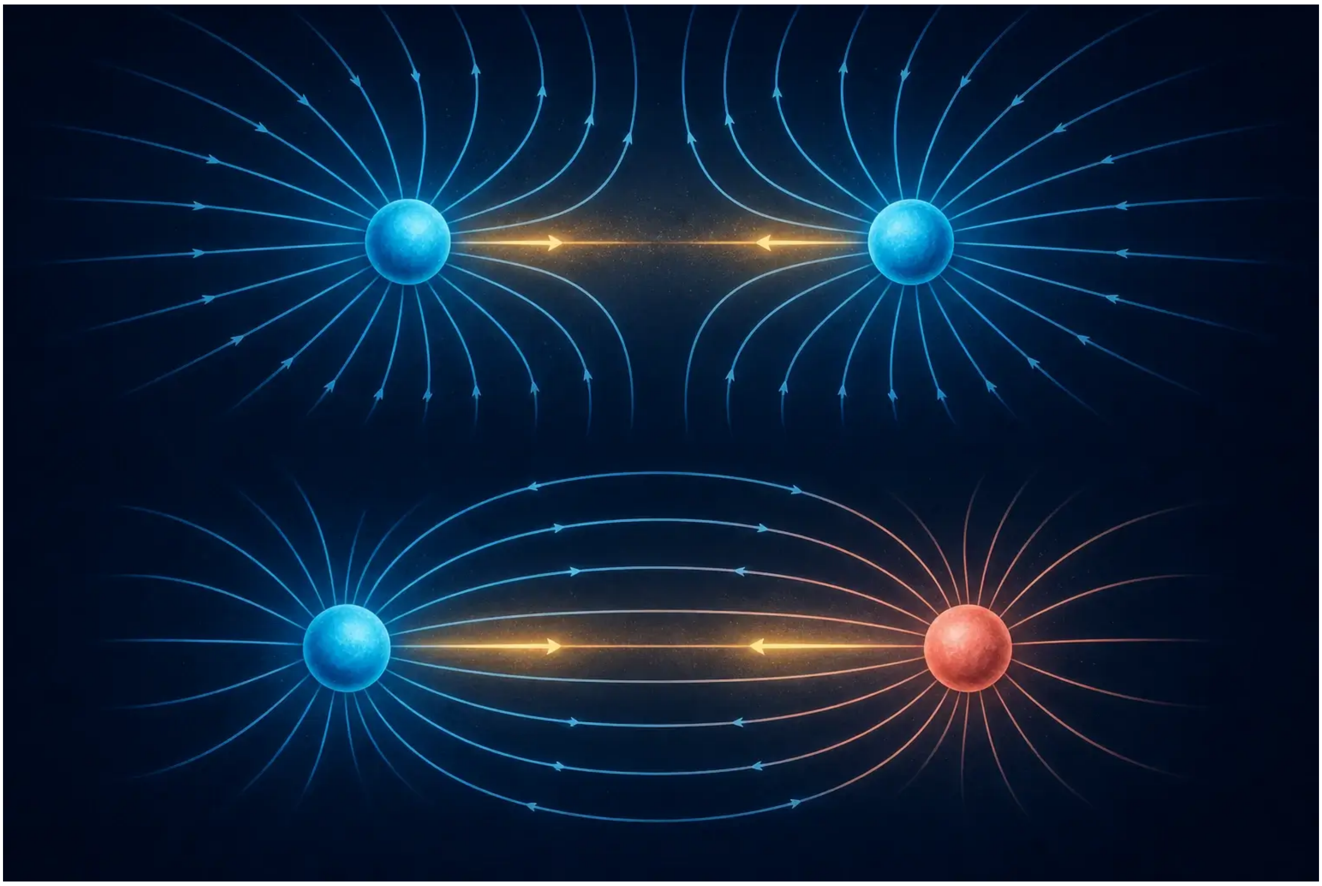
SUBJECT	LEVEL	INCLUDES
Physics	High school and early college	Practice set, answer key, revision sheet

Read this note online, with updates: gauravtiwari.org/study-notes/coulombs-law

INSIDE

Coulomb's law gives the electrostatic force between two stationary point charges. The force grows with the product of the charges and falls with the square of their separation. Like charges repel, while opposite charges attract.

The magnitude formula is simple, but sign and direction cause most errors. I recommend calculating a positive magnitude first, drawing the force direction from the charge signs, and then resolving components using your chosen coordinate axes.



Electric force acts along the line joining two charges. Its magnitude falls with the square of their separation.

What Is Coulomb's Law?

For two point charges in vacuum separated by distance r, the force magnitude is

$$F = k_{\mathrm{e}} \frac{|q_1 q_2|}{r^2}$$

Coulomb's law itself has no single SI unit. In $F = k_{\mathrm{e}}|q_1 q_2|/r^2$, force F is measured in newtons (N), charge in coulombs (C), distance in metres (m), and Coulomb's constant k_{e} in $\mathrm{N\,m^2C^{-2}}$. A question asking for the

SI unit of Coulomb's law usually means the unit of Coulomb's constant.

where $k_e = 1/(4\pi\varepsilon_0) \approx 8.9875517923 \times 10^9\ \mathrm{N\,m^2C^{-2}}$. The force acts along the line joining the charges.

SYMBOL	MEANING	SI UNIT
F	Force magnitude	newton (N)
q_1, q_2	Electric charges	coulomb (C)
r	Distance between charge positions	metre (m)
ε_0	Vacuum permittivity	farad per metre (F/m)
k_e	Coulomb constant	$\mathrm{N\,m^2C^{-2}}$

The OpenStax University Physics treatment of Coulomb's law gives the same inverse-square relation and develops superposition with vectors.

Direction and the Vector Form

The force on charge 2 due to charge 1 points along the unit vector from charge 1 to charge 2. A compact vector form is

$$\mathbf{F}_{2\leftarrow 1} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\mathbf{r}_2 - \mathbf{r}_1|^3}(\mathbf{r}_2 - \mathbf{r}_1)$$

The product $q_1 q_2$ carries the attraction or repulsion sign. If it is positive, the force on charge 2 points away from charge 1. If it is negative, the force points toward charge 1. Newton's third law gives $\mathbf{F}_{1\leftarrow 2} = -\mathbf{F}_{2\leftarrow 1}$.

- **Positive-positive:** each force points away from the other charge.
- **Negative-negative:** the charges also repel.
- **Positive-negative:** each force points toward the other charge.
- **Zero charge:** the electrostatic force from this pair is zero.

Worked Example: Two Point Charges

Place $q_1 = +2.0\,\mu\mathrm{C}$ and $q_2 = -3.0\,\mu\mathrm{C}$ at a separation of $0.50\,\mathrm{m}$. The force magnitude is

$$F = (8.99 \times 10^9)\frac{(2.0 \times 10^{-6})(3.0 \times 10^{-6})}{(0.50)^2} \approx 0.216\,\mathrm{N}$$

The force is attractive because the charges have opposite signs. The force on q_1 points toward q_2, and the equal-magnitude force on q_2 points toward q_1.

The Inverse-Square Dependence

Doubling the separation makes the force one quarter as large. Tripling it makes the force one ninth as large. This geometric dilution is the same distance dependence that appears in the field of a point source and in Newton's ideal point-mass gravity.

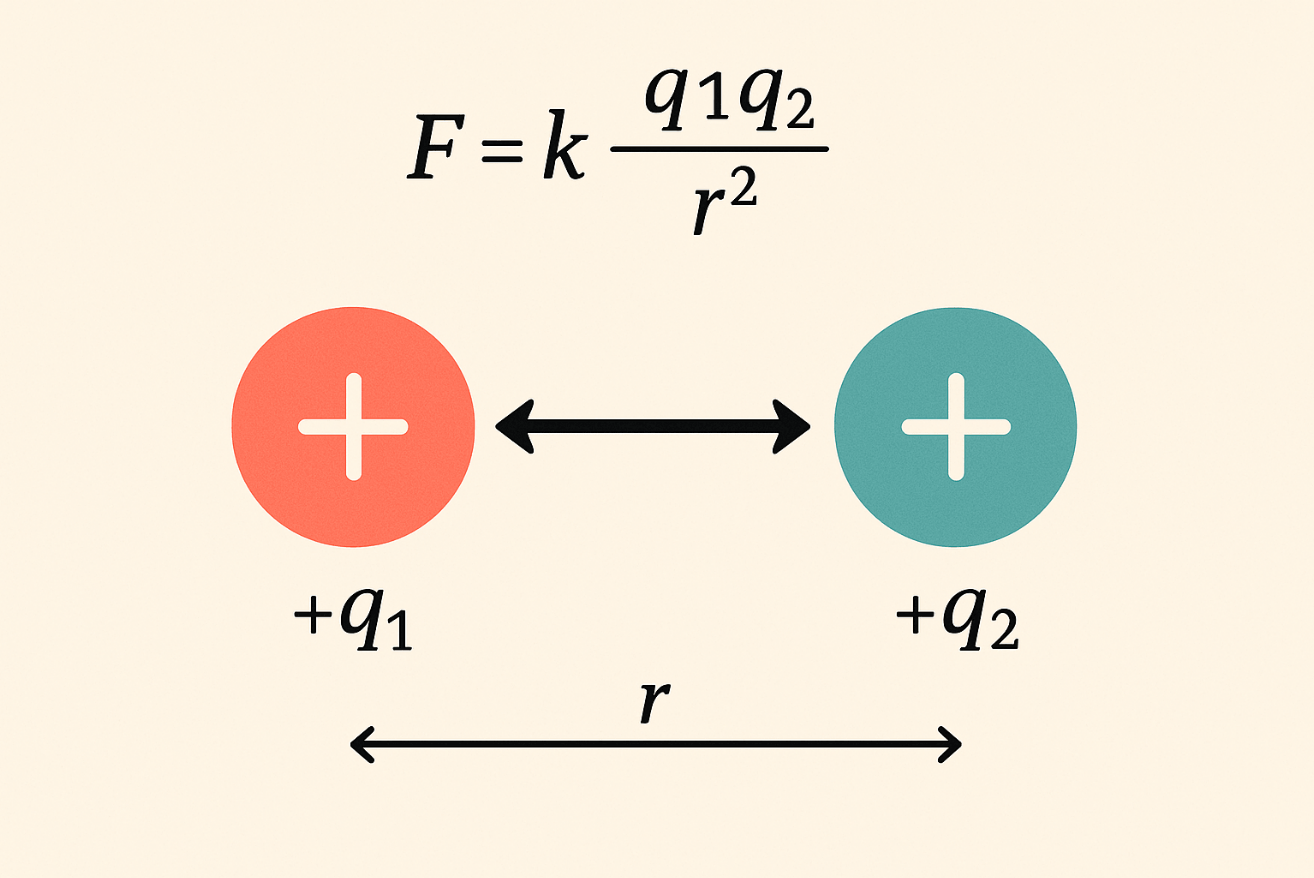
NEW SEPARATION	FORCE COMPARED WITH ORIGINAL
$r/2$	$4F$
$2r$	$F/4$
$3r$	$F/9$
$10r$	$F/100$

Do not confuse an inverse-square law with exponential decay. The force never reaches exactly zero at a finite distance in the ideal vacuum model, although it can become negligible compared with other effects.

Superposition for More Than Two Charges

Coulomb's law applies to each pair, and the total force is the vector sum. Other charges do not switch off a pair interaction. They add more force vectors.

$$\mathbf{F}_{\text{net on } i} = \sum_{j \neq i} \frac{1}{4\pi\varepsilon_0} \frac{q_i q_j}{|\mathbf{r}_i - \mathbf{r}_j|^3} (\mathbf{r}_i - \mathbf{r}_j)$$



Coulomb's law - the electrostatic force between two point charges is proportional to the product of the charges and inversely proportional to the square of the distance between them.

1. Draw the position of every charge and define axes.
2. Choose the charge whose net force you need.
3. Calculate the force from each other charge separately.
4. Resolve every force into x and y components.
5. Add components, then find the resultant magnitude and angle.

Three-Charge Symmetry Check

If two equal positive charges sit the same distance to the left and right of a third positive charge, their forces on the middle charge cancel. Each pair force is nonzero; the vector sum is zero. This is equilibrium by symmetry, not absence of electric interaction.

Electric Field and Coulomb's Law

The electric field is force per unit positive test charge. For a point source charge Q

$$\mathbf{E} = \frac{\mathbf{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}}$$

Once the field is known, any charge q placed there feels $\mathbf{F} = q\mathbf{E}$. A negative test charge feels force opposite the field direction. The field belongs to the source distribution; the force also depends on the test charge.

For highly symmetric continuous charge distributions, Gauss's law is often faster than adding contributions directly.

Coulomb's Law in Matter

In a uniform linear dielectric, the ideal macroscopic formula becomes

$$F = \frac{1}{4\pi\varepsilon}\frac{|q_1 q_2|}{r^2}, \qquad \varepsilon = \varepsilon_r\varepsilon_0$$

Polarization usually reduces the field produced by free charges compared with vacuum. But writing $F_{\text{vacuum}}/\varepsilon_r$ is only safe for a homogeneous, isotropic, linear medium in the relevant regime. Interfaces, nonlinear materials, and microscopic separations require more careful field and material models.

From Point Charges to Continuous Charge

For an extended object, divide the charge into elements dq, write the vector field contribution, and integrate over the distribution.

$$d\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{R^2}\hat{\mathbf{R}}, \qquad \mathbf{E} = \int d\mathbf{E}$$

The point-charge formula is also an excellent approximation outside a spherically symmetric charge distribution, or whenever the object's size is much smaller than the observation distance. It is not accurate near an irregularly shaped extended charge.

Common Mistakes

- **Using microcoulombs as coulombs:** $1\,\mu\text{C} = 10^{-6}\,\text{C}$.
- **Using diameter instead of center-to-center separation:** r is the distance between charge positions.
- **Keeping the algebraic sign inside the magnitude formula:** use absolute values for magnitude and determine direction separately.
- **Adding magnitudes instead of vectors:** forces in different directions require components.
- **Using Coulomb's law for moving charges without qualification:** magnetic fields and retardation can matter.
- **Applying a single dielectric constant across interfaces:** material boundaries change the field pattern.

Related Physics Guides

Use these next if you want to connect this result with the surrounding physics:

- Gauss's law and electric flux
- The four fundamental forces
- Symmetry in physical laws

Key Takeaways

- Coulomb's law has magnitude $F = k_e|q_1 q_2|/r^2$.
- Like charges repel and opposite charges attract.
- The force is a vector directed along the line joining the charges.

- For many charges, add the pairwise forces by superposition.
- In matter, permittivity and material geometry affect the field.

Calculate magnitude, draw direction, and add vectors. That three-step habit prevents nearly every beginner error.

Practice Questions

Work each question before reading its solution. The set runs from direct recall and substitution to the applied questions that exams actually use to separate grades.

Q1. Two charges of 1 μC each sit 10 cm apart. Find the force between them.

SOLUTION

$F = k\frac{q_1 q_2}{r^2} = \frac{9\times10^9\times(10^{-6})^2}{(0.1)^2} = \frac{9\times10^{-3}}{0.01} = 0.9$ N. Nearly a full newton from 2 microscopic charge amounts at hand-width distance: the electric force is enormous.

Q2. The distance between 2 charges doubles. What happens to the force?

SOLUTION

The inverse-square law divides it by 2^2: the force drops to a quarter. Halving the distance would quadruple it. Distance enters squared, so it dominates every estimate.

Q3. How does the force differ between like and unlike charges?

SOLUTION

The magnitude is identical for the same $|q_1 q_2|$ and r; only the direction changes. Like charges repel along the line joining them; unlike charges attract along the same line. Both charges feel equal and opposite forces, Newton's third law included.

Q4. Compute the electric force between the proton and electron in hydrogen ($r = 5.3\times10^{-11}$ m).

SOLUTION

$F = \frac{9\times10^9\times(1.6\times10^{-19})^2}{(5.3\times10^{-11})^2} = \frac{2.3\times10^{-28}}{2.8\times10^{-21}} \approx 8.2\times10^{-8}$ N. Tiny in absolute terms, but acting on an electron of mass 9.1×10^{-31} kg it produces an acceleration of about 10^{22} m/s^2.

Q5. For the same hydrogen atom, compare the electric force with the gravitational attraction between the 2 particles.

SOLUTION

$F_g = G\frac{m_e m_p}{r^2} = \frac{6.67\times10^{-11}\times9.1\times10^{-31}\times1.67\times10^{-27}}{2.8\times10^{-21}} \approx 3.6\times10^{-47}$ N. The ratio is about 2×10^{39}: electricity beats gravity by 39 orders of magnitude. Gravity rules planets only because bulk matter is almost perfectly charge-neutral.

Q6. Charges $+q$ sit at $x = 0$ and $x = 1$ m. Where does a test charge feel zero net force?

SOLUTION

At the midpoint, $x = 0.5$ m. Both repulsions have equal magnitude there and point in opposite directions, canceling exactly. Symmetry answers before algebra does; for unequal charges the balance point shifts toward the weaker one.

Q7. Charges $+2\,\mu$C, $-1\,\mu$C, and $+2\,\mu$C sit at $x = 0, 0.5$, and 1 m. Find the net force on the middle charge.

SOLUTION

The left $+2\,\mu$C attracts the middle charge leftward with $F = \dfrac{9 \times 10^9 \times 2 \times 10^{-12}}{0.25} = 0.072$ N. The right $+2\,\mu$C attracts it rightward with the same 0.072 N by symmetry. The pulls cancel: net force zero. Superposition means summing forces as vectors, and here the vectors kill each other.

Q8. Find the electric field 0.3 m from a $5\,\mu$C point charge.

SOLUTION

$E = k\dfrac{q}{r^2} = \dfrac{9 \times 10^9 \times 5 \times 10^{-6}}{0.09} = 5 \times 10^5$ N/C, pointing radially away. The field is force per unit charge: any charge q placed there feels $F = qE$.

Q9. How are k and ε_0 related?

SOLUTION

$k = \dfrac{1}{4\pi\varepsilon_0} \approx 9 \times 10^9$ N·m^2/C^2, with $\varepsilon_0 = 8.85 \times 10^{-12}$ in SI units. The 4π version makes later formulas cleaner: Gauss's law, capacitance, and field equations all prefer ε_0.

Q10. Why an inverse-square law? Give the geometric picture.

SOLUTION

Picture the influence of a point charge spreading uniformly through space. At radius r it crosses a sphere of area $4\pi r^2$, so the same total influence spreads over an area growing as r^2, diluting the intensity as $1/r^2$. Gravity and light obey the same geometry, which is why all 3 share the same law shape.

Answer Key

QUICK ANSWERS

Q1 0.9 N **Q2** drops to $F/4$ **Q3** same magnitude; repel vs attract **Q4** 8.2×10^{-8} N **Q5** electric force larger by $\sim 10^{39}$

Q6 the midpoint, $x = 0.5$ m **Q7** zero **Q8** 5×10^5 N/C **Q9** $k = 1/(4\pi\varepsilon_0)$ **Q10** influence spreads over spheres of area $4\pi r^2$

Revision Sheet

The law

$F = k\frac{|q_1 q_2|}{r^2}$, $k = 9 \times 10^9\ \mathrm{N{\cdot}m^2/C^2}$, along the joining line.

Superposition

Net force = vector sum of pairwise Coulomb forces. Symmetry first, algebra second.

Directions

Like charges repel, unlike attract; the pair feels equal and opposite forces.

The field

$E = kq/r^2$, force per unit charge; then $F = qE$.

Scaling

Double the distance: $F/4$. Double one charge: $2F$.

Perspective

Electric force between proton and electron beats their gravity by $\sim 10^{39}$.

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