THERMAL PHYSICS

Boltzmann Constant: Meaning, Value, and Applications

What k converts, the kT energy scale, gas laws per molecule, entropy, and a 10-question practice set with full solutions.

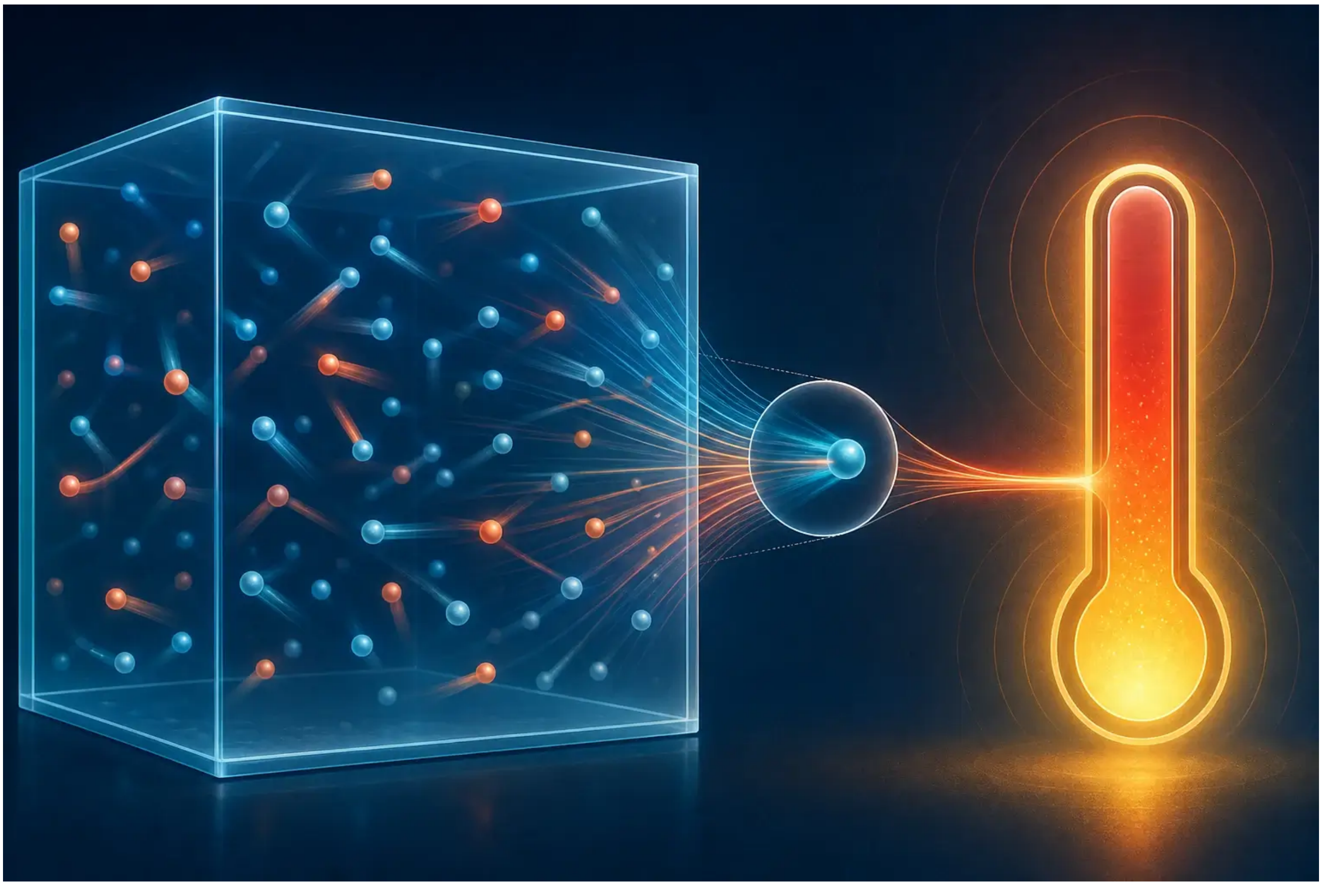
SUBJECT	LEVEL	INCLUDES
Physics	High school and early college	Practice set, answer key, revision sheet

Read this note online, with updates: gauravtiwari.org/study-notes/boltzmann-constant

INSIDE

The **Boltzmann constant** converts thermodynamic temperature into an energy scale for individual particles. Its exact SI value is $k_B = 1.380649 \times 10^{-23}\ \mathrm{J\,K^{-1}}$. Multiply it by temperature and you get a characteristic thermal energy.

That tiny number is the bridge between the microscopic and macroscopic worlds. It links molecular motion to temperature, counts microstates through entropy, weights energy states through probability, and connects the particle form of the ideal-gas law to the molar form.

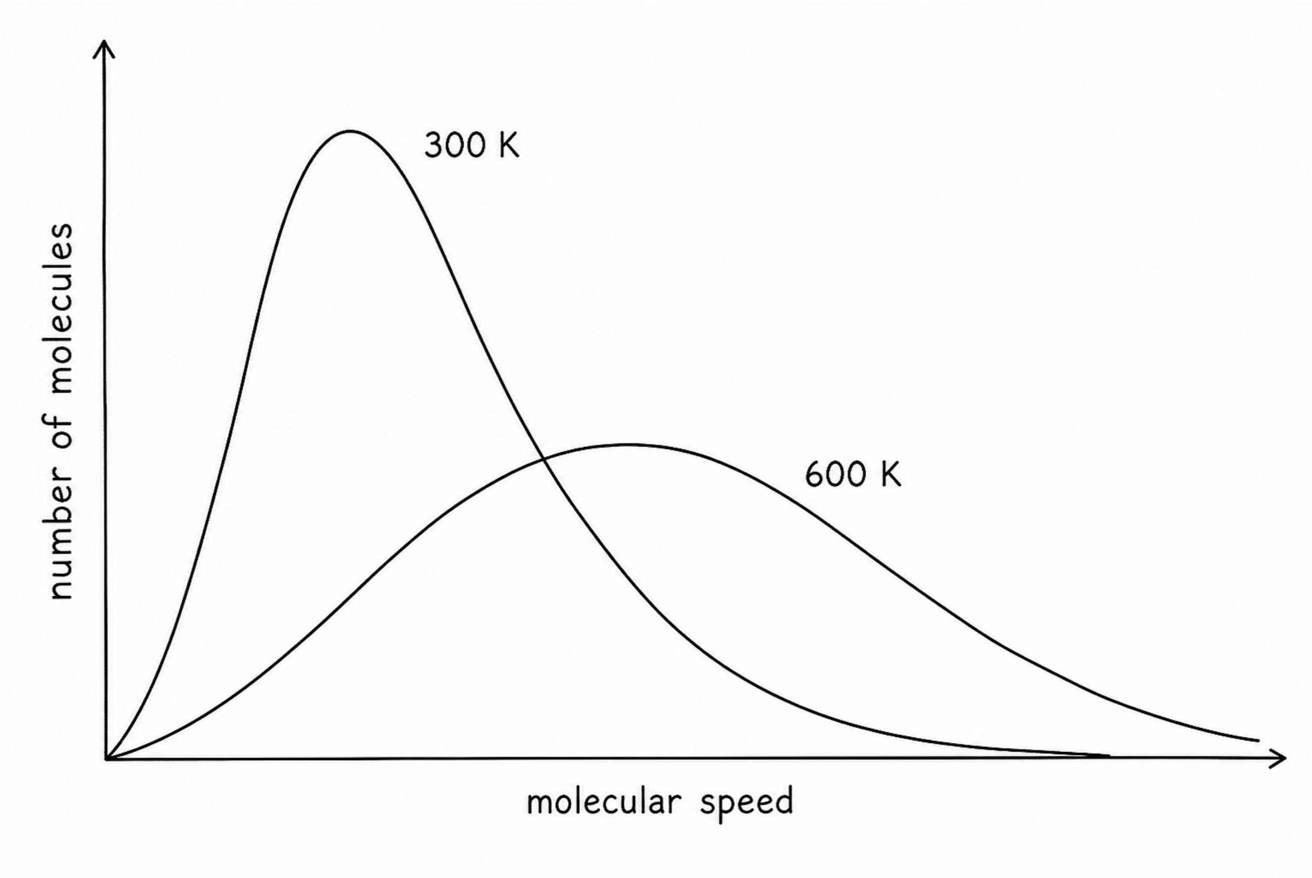


Boltzmann's constant converts the temperature scale into an energy scale for individual particles.

If you want to check a value before working it by hand, use the Boltzmann constant calculator. It keeps the unit conversion beside the result, which is where most avoidable mistakes happen.

What Is the Boltzmann Constant?

The Boltzmann constant is a universal proportionality constant that relates energy to absolute temperature. One kelvin corresponds to an energy increment of 1.380649×10^{-23} joule per particle on the scale $k_B T$.



Molecular speeds at 2 temperatures: heating shifts the peak right and flattens the curve, but the area (the molecule count) stays fixed.

$$k_B = 1.380649 \times 10^{-23}\ \mathrm{J\ K^{-1}} \quad \text{(exact)}$$

The value is exact because the 2019 revision of the International System of Units fixed k_B by definition. The kelvin is now defined through the Boltzmann constant rather than through the triple point of water. NIST explains the change in its kelvin and Boltzmann constant guide.

FORM	VALUE	USEFUL CONTEXT
k_B	$1.380649 \times 10^{-23}\ \mathrm{J\ K^{-1}}$	SI energy per particle
k_B	$8.617333262 \times 10^{-5}\ \mathrm{eV\ K^{-1}}$	Atomic and semiconductor energy scales
$R = N_A k_B$	$8.314462618\ \ldots\ \mathrm{J\ mol^{-1} K^{-1}}$	Energy per mole

The Thermal Energy Scale kBT

The product $k_B T$ is not the energy of every particle. It is the energy scale that decides whether thermal fluctuations can populate states separated by an energy gap ΔE. When $\Delta E \ll k_B T$, thermal access is easy. When $\Delta E \gg k_B T$, it is strongly suppressed.

Room-Temperature Example

At $T = 300\,\mathrm{K}$

$$k_B T = (1.380649 \times 10^{-23})(300) = 4.141947 \times 10^{-21}\,\mathrm{J}$$

$$k_B T \approx 0.02585\,\mathrm{eV}$$

The electron-volt form is especially useful in atomic physics and electronics. A 1 eV energy gap is about 38.7 times $k_B T$ at 300 K, so ordinary room-temperature fluctuations rarely cross it without another source of energy.

Boltzmann Constant and Temperature

Temperature is not the total energy of a body. It is a thermodynamic variable tied to how entropy changes with energy. In statistical mechanics, k_B sets the units so microscopic energy changes and macroscopic temperature use the same probability laws.

For a monatomic ideal gas in thermal equilibrium, the mean translational kinetic energy per particle is

$$\langle K \rangle = \frac{3}{2} k_B T$$

At 300 K this mean is about 6.21×10^{-21} J, or 0.0388 eV. Individual molecules have a distribution of speeds and energies; they do not all carry exactly the mean value.

Boltzmann Constant and Entropy

Boltzmann's famous relation is

$$S = k_{\mathrm{B}} \ln \Omega$$

Here Ω is the number, or appropriate measure, of microscopic states compatible with the macrostate. The logarithm makes entropy additive when independent multiplicities multiply. The macrostates and microstates guide works through that counting logic.

In more general statistical mechanics, entropy can be written $S = -k_{\mathrm{B}} \sum_i p_i \ln p_i$. If all Ω accessible microstates have equal probability $p_i = 1/\Omega$, the formula reduces to $S = k_{\mathrm{B}} \ln \Omega$.

The Boltzmann Factor

For a system exchanging energy with a heat reservoir at temperature T, a state's relative statistical weight contains the Boltzmann factor

$$e^{-E/(k_{\mathrm{B}}T)}$$

Two states separated by ΔE have the population ratio $P_2/P_1 = \exp[-\Delta E/(k_{\mathrm{B}}T)]$, apart from degeneracy factors. Raising temperature makes high-energy states less strongly suppressed.

Example: A 0.10 eV Energy Gap

At 300 K, $k_{\mathrm{B}}T \approx 0.02585$ eV. Ignoring degeneracy

$$\frac{P_2}{P_1} = e^{-0.10/0.02585} \approx 2.09 \times 10^{-2}$$

The upper state has about 2.1% of the lower state's weight. At higher temperature that ratio rises; at lower temperature it falls.

Boltzmann Constant in the Ideal-Gas Law

The particle form of the ideal-gas equation is

$$PV = Nk_\mathrm{B}T$$

where N is the number of molecules. The molar form is $PV = nRT$. Since $N = nN_\mathrm{A}$, consistency requires $R = N_\mathrm{A}k_\mathrm{B}$. Use k_B for individual particles and R for moles.

The ideal gas law guide shows how to choose units and solve pressure-volume-temperature problems.

Where the Constant Appears

TOPIC	RELATION	WHAT KB DOES
Thermal energy	$E_\mathrm{thermal} \sim k_\mathrm{B}T$	Turns temperature into energy per particle
Ideal gas	$PV = Nk_\mathrm{B}T$	Connects pressure-volume work to particle count
Entropy	$S = k_\mathrm{B} \ln \Omega$	Gives microscopic counting macroscopic entropy units
Canonical probability	$p_i \propto e^{-E_i/(k_\mathrm{B}T)}$	Sets thermal suppression of high-energy states
Equipartition	$\frac{1}{2}k_\mathrm{B}T$ per quadratic degree	Sets mean energy contribution under classical conditions
Semiconductors	$e^{-E/(k_\mathrm{B}T)}$	Controls thermal carrier activation and diode behaviour

Common Confusions

- **Boltzmann constant vs gas constant:** k_B is per particle; R is per mole.
- **Boltzmann constant vs Stefan-Boltzmann constant:** k_B links temperature and particle energy; σ appears in blackbody power $j^\star = \sigma T^4$.
- **kBT vs exact particle energy:** $k_\mathrm{B}T$ is a scale, not a claim that every particle has one identical energy.
- **Kelvin vs Celsius:** statistical formulas require absolute temperature.
- **Classical vs quantum limits:** equipartition can fail when quantum energy spacings are not small compared with $k_\mathrm{B}T$.

Related Physics Guides

Use these next if you want to connect this result with the surrounding physics:

- Macrostates and microstates
- Ideal gas law
- Laws of thermodynamics

Key Takeaways

- The Boltzmann constant has the exact value $1.380649 \times 10^{-23}\ \mathrm{J\,K^{-1}}$.
- The product $k_\mathrm{B}T$ is the characteristic thermal energy per particle.
- Entropy and multiplicity are related by $S = k_\mathrm{B} \ln \Omega$.
- The Boltzmann factor $e^{-E/(k_\mathrm{B}T)}$ weights energy states in the canonical ensemble.
- The gas constants satisfy $R = N_\mathrm{A} k_\mathrm{B}$.

Whenever you see k_B, read it as the conversion rate between kelvin and microscopic energy.

Practice Questions

Work each question before reading its solution. The set runs from direct recall and substitution to the applied questions that exams actually use to separate grades.

Q1. State the value of the Boltzmann constant and what it converts between.

SOLUTION

$k = 1.380649 \times 10^{-23}$ J/K, exact by definition since the 2019 SI reform. It converts temperature into energy at the scale of single particles: kelvin on one side, joules per molecule on the other.

Q2. Show that $R = N_A k$ reproduces the gas constant.

SOLUTION

$N_A k = 6.022 \times 10^{23} \times 1.381 \times 10^{-23} = 8.314\ \mathrm{J/(mol{\cdot}K)}$, which is R. The gas constant is just the Boltzmann constant scaled from 1 molecule to 1 mole.

Q3. Find the average translational kinetic energy of a gas molecule at 300 K.

SOLUTION

$\langle E \rangle = \frac{3}{2}kT = 1.5 \times 1.381 \times 10^{-23} \times 300 = 6.2 \times 10^{-21}$ J. Every ideal-gas molecule averages this, regardless of its mass: heavier molecules just move slower to carry the same energy.

Q4. Compute kT at room temperature (300 K) in joules and electronvolts.

SOLUTION

$kT = 1.381 \times 10^{-23} \times 300 = 4.14 \times 10^{-21}$ J. Dividing by 1.602×10^{-19} J/eV gives 0.026 eV, about $\frac{1}{40}$ eV. This number is the thermal energy yardstick for chemistry, semiconductors, and biology alike.

Q5. Use $pV = NkT$ to count the molecules in 1 liter of air at 1 atm and 300 K.

SOLUTION

$N = \frac{pV}{kT} = \frac{101{,}325 \times 0.001}{4.14 \times 10^{-21}} \approx 2.4 \times 10^{22}$ molecules. Twenty-four thousand billion billion molecules in a soda bottle of air: the constant's tiny size is the flip side of matter's enormous granularity.

Q6. Find the rms speed of a nitrogen molecule ($m = 4.65 \times 10^{-26}$ kg) at 300 K.

SOLUTION

$v_{rms} = \sqrt{3kT/m} = \sqrt{1.243 \times 10^{-20}/4.65 \times 10^{-26}} = \sqrt{2.67 \times 10^5} \approx 517$ m/s. The air in this room is a hail of molecules moving faster than sound; pressure is their collective drumming.

Q7. State Boltzmann's entropy formula and what W counts.

SOLUTION

$S = k \ln W$, where W counts the microstates: the number of distinct molecular arrangements consistent with what you can measure macroscopically. Entropy is literally a count of possibilities, scaled by k into thermodynamic units. The formula is carved on Boltzmann's gravestone.

Q8. How does the SI define the kelvin since 2019?

SOLUTION

By fixing $k = 1.380649 \times 10^{-23}$ J/K exactly and defining 1 kelvin as the temperature change that shifts kT by that many joules. Temperature is now defined through energy, not through water's triple point, which is measured rather than defined.

Q9. Why does the factor $e^{-E/kT}$ appear throughout physics and chemistry?

SOLUTION

It is the Boltzmann factor: the relative probability that thermal jostling assembles energy E in one place. Reaction rates, vapor pressures, semiconductor currents, and even the atmosphere's thinning with altitude all follow it, because each asks the same question: how often does random thermal motion supply this much energy? The ratio E/kT decides everything.

Q10. A chemical bond stores roughly 4 eV. Compare this with kT at room temperature and explain the consequence.

SOLUTION

4 eV/0.026 eV $\approx$ 150, so a bond stores about 150 times the typical thermal kick. The Boltzmann factor e^{-150} is about 10^{-65}: thermal motion essentially never breaks a bond by chance. That is why molecules survive at room temperature, and why heating, catalysts, or enzymes are needed to make chemistry happen on human timescales.

Answer Key

QUICK ANSWERS

Q1 1.380649×10^{-23} J/K **Q2** $6.022 \times 10^{23} \times 1.381 \times 10^{-23} \approx 8.314$ **Q3** 6.2×10^{-21} J **Q4** 4.14×10^{-21} J = 0.026 eV **Q5** about 2.4×10^{22}

Q6 about 517 m/s **Q7** $S = k \ln W$; microstates **Q8** via the fixed exact value of k **Q9** it is the probability of borrowing energy E from heat **Q10** bonds hold ~150 kT, so thermal breakage is negligible

Revision Sheet

The constant

$k = 1.380649 \times 10^{-23}$ J/K, exact. Bridges kelvin and joules per particle.

Per-molecule gas law

$pV = NkT$

Mole bridge

$R = N_A k = 8.314$ J/(mol·K).

Speeds

$v_{rms} = \sqrt{3kT/m}$; N_2 at 300 K: about 517 m/s.

Energy scales

$\langle E \rangle = \frac{3}{2}kT$; at 300 K, $kT \approx 0.026$ eV $\approx \frac{1}{40}$ eV.

Entropy and probability

$S = k \ln W$; Boltzmann factor $e^{-E/kT}$ sets how often thermal motion supplies energy E.

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