SEQUENCES

Arithmetic Progression

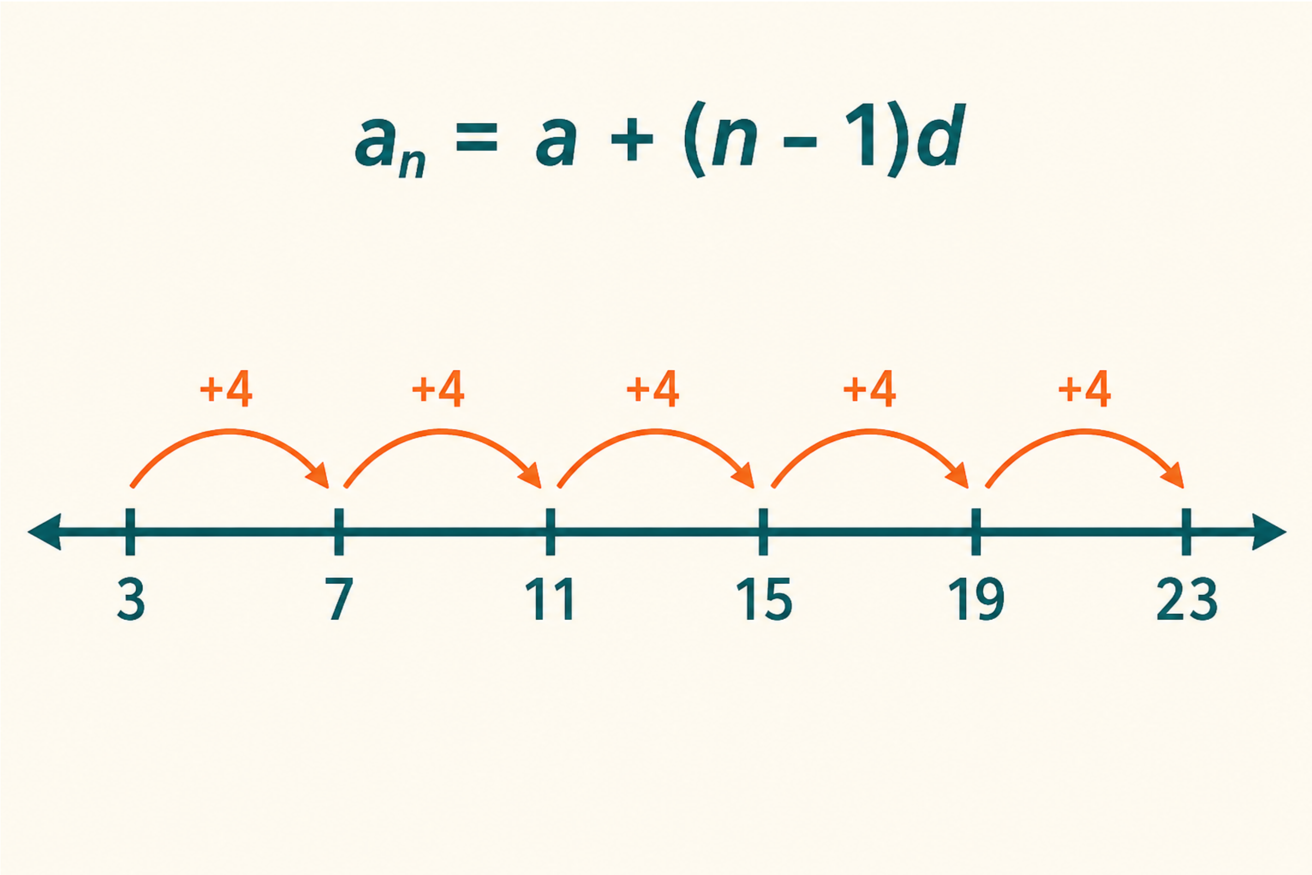
Constant-step sequences: the nth term, the sum formulas, and the classic problems, with a 10-question practice set and full solutions.

SUBJECT	LEVEL	INCLUDES
Mathematics	High school and early college	Practice set, answer key, revision sheet

Read this note online, with updates: gauravtiwari.org/study-notes/arithmetic-progression

INSIDE

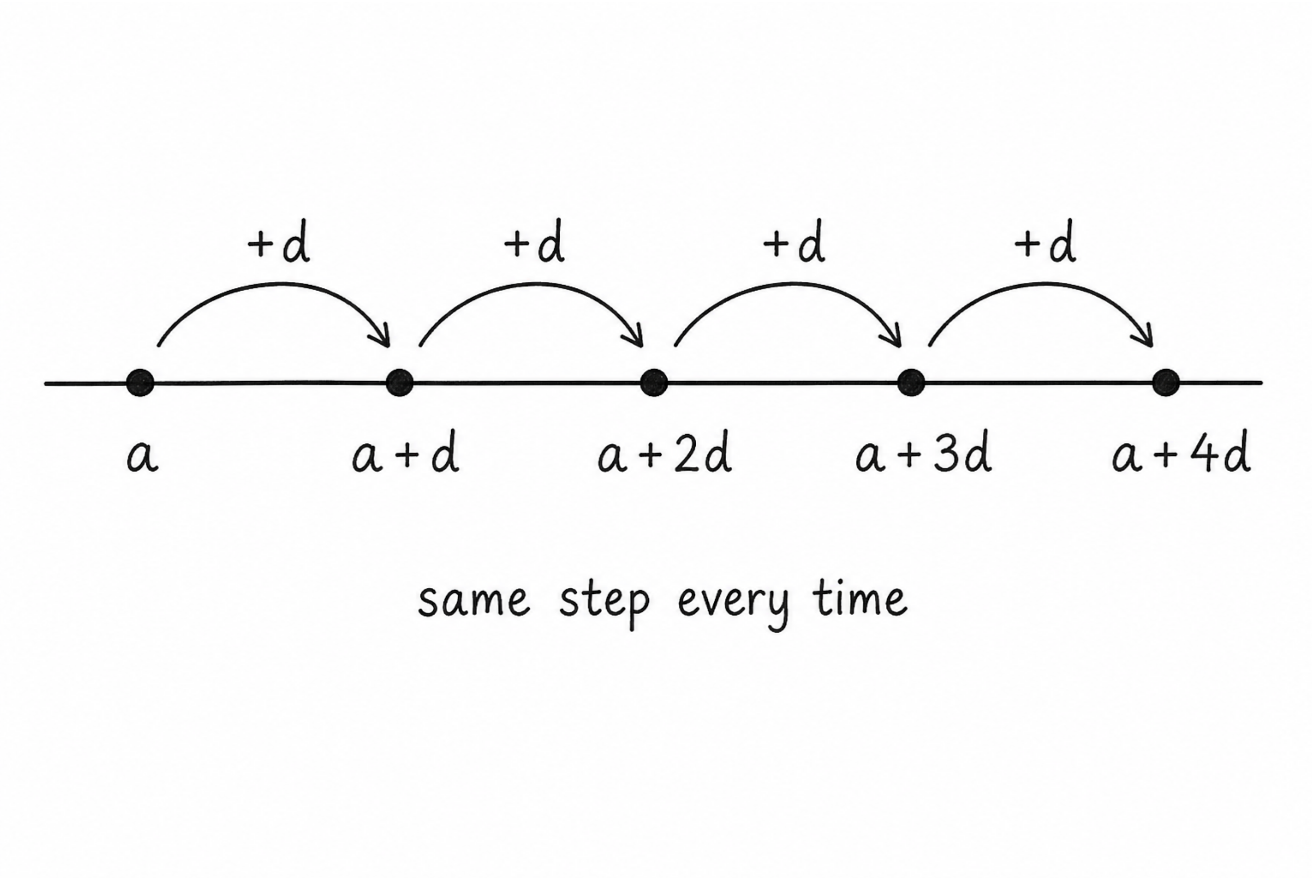
An arithmetic progression (AP) is a sequence in which each term differs from the previous one by a fixed amount called the common difference. Unlike a geometric progression where you multiply by a constant ratio, an AP grows by repeated addition. APs describe situations with linear, steady change: equal monthly savings, stair steps of fixed height, the days in successive months of a year, simple-interest growth, and rows of seats that gain a fixed number from front to back.



An arithmetic progression has a constant difference between consecutive terms. General term: a_n = a + (n-1)d.

Definition and General Term

An arithmetic progression has the form: $a, a + d, a + 2d, a + 3d, \ldots$, where a is the first term and d is the common difference. The n-th term is:



An AP on the number line: the same jump, +d, every single step.

$$a_n = a + (n-1)\,d$$

A sequence is an AP precisely when the difference between consecutive terms is constant: $a_2 - a_1 = a_3 - a_2 = \cdots = d$.

Worked Examples

- **3, 7, 11, 15, 19, …:** $a = 3$, $d = 4$. 20th term: $3 + 19 \cdot 4 = 79$.
- **100, 95, 90, 85, …:** $a = 100$, $d = -5$. Decreasing AP. 21st term: $100 + 20 \cdot (-5) = 0$.
- **$\frac{1}{2}, 1, \frac{3}{2}, 2, \ldots$:** $a = \frac{1}{2}$, $d = \frac{1}{2}$. 15th term: $\frac{1}{2} + 14 \cdot \frac{1}{2} = \frac{15}{2} = 7.5$.

Sum of an AP

The sum of the first n terms of an AP is:

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a + a_n)$$

The second form has an elegant geometric interpretation: the sum equals the number of terms times the average of the first and last terms.

The Story of Gauss

Carl Friedrich Gauss, age 7 or 8, was reportedly told by his teacher to sum the integers from 1 to 100, a task meant to keep the class occupied. Gauss noticed that pairing 1 + 100, 2 + 99, 3 + 98, all equal 101, and there are 50 such pairs, so the total is 50 · 101 = 5050. He had the answer almost instantly. The story is the most famous illustration of the AP sum formula.

Arithmetic Mean

In any AP, each term (except the first and last) is the arithmetic mean of its neighbors: $a_n = \frac{1}{2}(a_{n-1}+a_{n+1})$. The arithmetic mean of two numbers x and y is $(x + y)/2$, the most familiar 'average' in everyday usage.

AP vs GP: Side-by-Side

PROPERTY	ARITHMETIC PROGRESSION	GEOMETRIC PROGRESSION
Operation between terms	Addition (constant d)	Multiplication (constant r)
nth term	$a + (n-1)d$	$a \cdot r^{(n-1)}$
Sum of n terms	$n/2 \cdot [2a + (n-1)d]$	$a \cdot (1 - r^n)/(1 - r)$
Type of growth	Linear	Exponential
Sequence example	3, 7, 11, 15...	3, 12, 48, 192...

Applications

- **Simple interest.** Money invested at simple interest grows arithmetically: P, P + Pr, P + 2Pr, P + 3Pr..., an AP with common difference Pr.
- **Salary increments.** A fixed annual raise of a set dollar amount produces an AP of yearly earnings.
- **Stadium seating.** Stadium rows often gain a fixed number of seats per row; the row counts form an AP.
- **Distance with constant deceleration.** An object decelerating uniformly covers distances in successive seconds that form an AP. (Galileo proved this geometrically.)
- **Counting and combinatorics.** The sum $1+2+\ldots+n$ is the count of handshakes between $n+1$ people, the number of edges in a complete graph, and the closed form $n(n+1)/2$.

Related study notes: Geometric Progression, Summation Notation, Fibonacci Sequence, Compound Interest.

Practice Questions

Work each question before reading its solution. The set runs from direct recall and substitution to the applied questions that exams actually use to separate grades.

Q1. For the AP 7, 11, 15, 19, ..., identify a and d, and find the 25th term.

SOLUTION

First term $a = 7$, common difference $d = 4$. $a_{25} = a + 24d = 7 + 96 = 103$. The nth term needs $n - 1$ steps, not n: off-by-one here is the single most common AP error.

Q2. Which term of the AP 3, 8, 13, ... equals 203?

SOLUTION

$a_n = 3 + 5(n-1) = 203$ gives $5(n-1) = 200$, so $n = 41$. The 41st term. Solving for n turns the term formula into a membership test: had n come out fractional, 203 would not belong to the sequence.

Q3. Sum the first 100 natural numbers, the way young Gauss reportedly did.

SOLUTION

Pair the ends: $1 + 100 = 101$, $2 + 99 = 101$, ..., making 50 pairs of 101: $S = 5050$. The general formula $S_n = \frac{n}{2}(a + l)$ is exactly this pairing written once for all.

Q4. Compute the sum of the first 20 terms of 5, 9, 13, ...

SOLUTION

$S_{20} = \frac{20}{2}[2(5) + 19(4)] = 10[10 + 76] = 860$. With no last term in hand, use $S_n = \frac{n}{2}[2a + (n-1)d]$; with the last term known, $\frac{n}{2}(a + l)$ is faster. Same formula, two outfits.

Q5. An AP has $a_5 = 22$ and $a_{12} = 57$. Find a and d.

SOLUTION

Subtract: $a_{12} - a_5 = 7d = 35$, so $d = 5$; then $a = 22 - 4(5) = 2$. Differences of terms leap straight to d because the a's cancel: always subtract the given equations first.

Q6. How many multiples of 7 lie between 100 and 500?

SOLUTION

First: 105; last: 497. From $497 = 105 + 7(n - 1)$: $n - 1 = 56$, so $n = 57$. Counting multiples is an AP with $d = 7$, and the endpoints must themselves be multiples before the formula applies.

Q7. The sum of 3 consecutive AP terms is 27 and their product is 585. Find them.

SOLUTION

Write them $m - d, m, m + d$: the sum gives $3m = 27$, $m = 9$. Product: $9(81 - d^2) = 585$, so $81 - d^2 = 65$, $d = \pm 4$. The terms are 5, 9, 13. Symmetric naming around the middle makes the sum trivial and the algebra quadratic instead of cubic.

Q8. A theater has 20 rows: 12 seats in the first, each row 3 more than the last. Find the seats in row 20 and the total capacity.

SOLUTION

Row 20: $12 + 19(3) = 69$ seats. Total: $S_{20} = \frac{20}{2}(12 + 69) = 810$. Auditoriums, log stacks, and staircase bricks are APs in the wild; the sum formula replaces 20 additions.

Q9. A loan is repaid in installments forming an AP: 1,000 in month 1, decreasing by 50 each month, until the payment reaches 100. How much is repaid in all?

SOLUTION

From $100 = 1000 - 50(n - 1)$: $n = 19$ payments. Sum: $\frac{19}{2}(1000 + 100) = 10{,}450$. Note the count first, then the sum: computing sums before pinning n is where these problems go wrong.

Q10. Insert 4 numbers between 8 and 33 so all 6 form an AP.

SOLUTION

Six terms means 5 gaps: $d = (33 - 8)/5 = 5$. The inserted numbers: 13, 18, 23, 28. Inserting k means between two values always gives $d = \frac{b-a}{k+1}$: gaps outnumber insertions by one.

Answer Key

QUICK ANSWERS

Q1 $a_{25} = 103$ **Q2** the 41st term **Q3** 5050 **Q4** 860 **Q5** $a = 2, d = 5$

Q6 57 **Q7** 5, 9, 13 **Q8** 69 seats; 810 total **Q9** 10,450 over 19 payments **Q10** 13, 18, 23, 28

Revision Sheet

Definition

Constant step: $a_{n+1} - a_n = d$. Sequence $a, a+d, a+2d, \ldots$

3 consecutive terms

Name them $m-d, m, m+d$: the sum collapses to $3m$.

nth term

$a_n = a + (n-1)d$: $n-1$ steps, not n.

Insertions

k means between a and b: $d = \frac{b-a}{k+1}$.

Sums

$S_n = \frac{n}{2}(a+l) = \frac{n}{2}[2a + (n-1)d]$. Gauss pairing in a formula.

Membership test

Solve $a_n = x$ for n: a whole-number n means x belongs.

About These Notes

This note is part of a free study-notes series on gauravtiwari.org covering mathematics, physics, chemistry, and biology: the concepts that keep deciding grades in high school, board, and entrance exams.

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