FLUID MECHANICS

Archimedes' Principle

Buoyancy from displaced fluid, floating and sinking, apparent weight, and a 10-question practice set with full solutions.

SUBJECT	LEVEL	INCLUDES
Physics	High school and early college	Practice set, answer key, revision sheet

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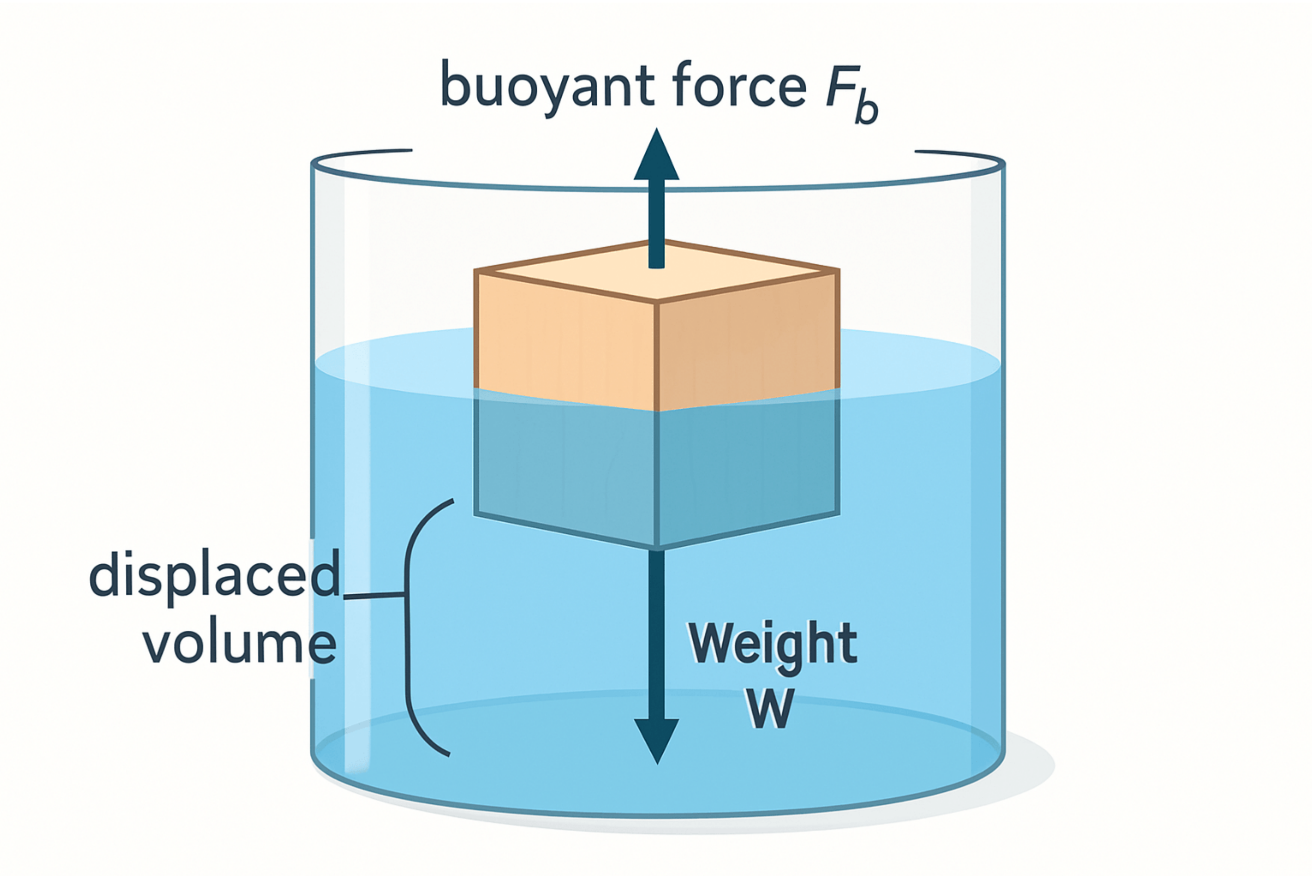
INSIDE

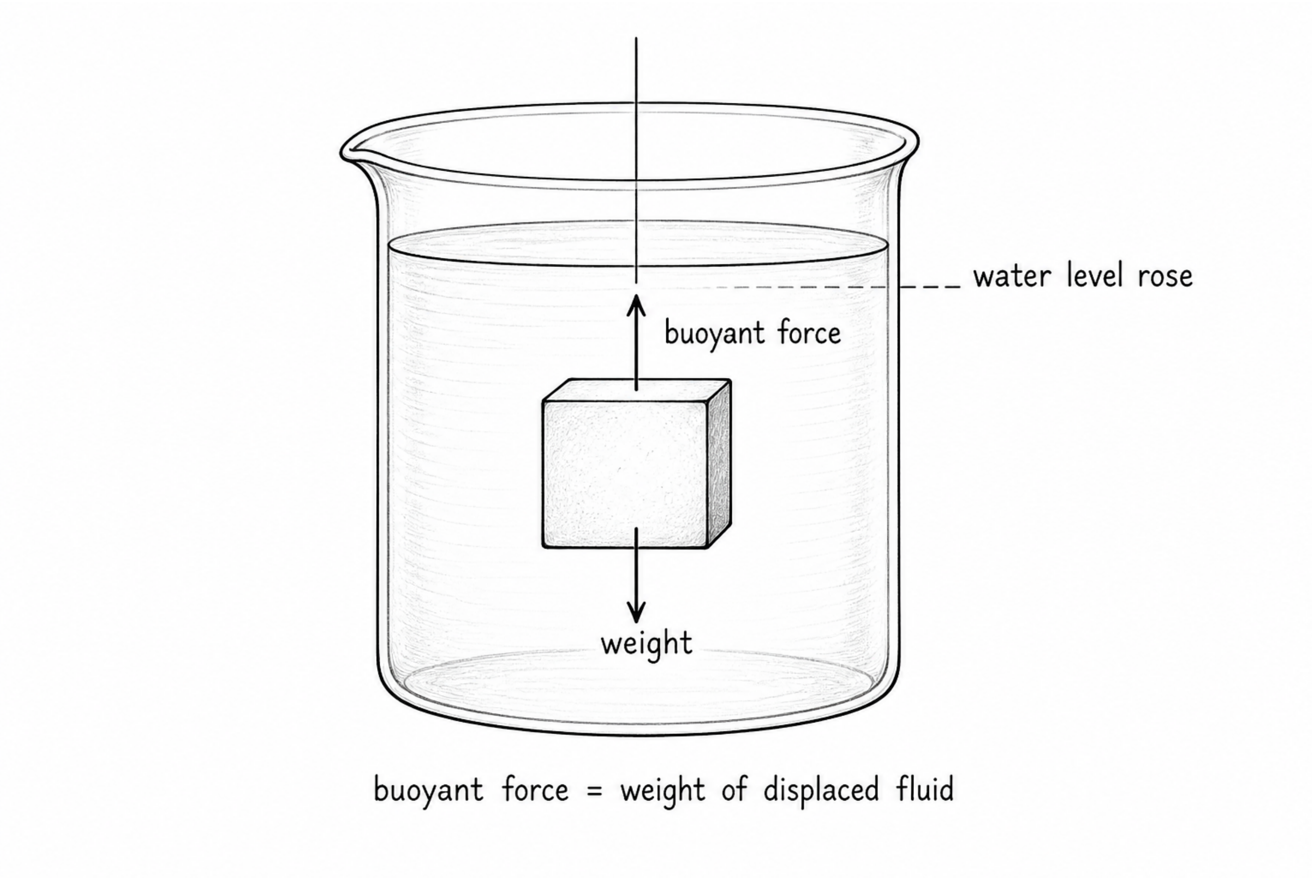
Archimedes' principle states that any object placed in a fluid experiences an upward buoyant force equal to the weight of the fluid it displaces. It explains why ships float, why you feel lighter in water, why hot-air balloons rise, and why submarines can sink and resurface at will. Discovered by the Greek mathematician Archimedes around 250 BCE, the principle is still the foundation of fluid statics, naval architecture, and density measurement.



Archimedes' principle: the buoyant force on a submerged object equals the weight of the fluid it displaces.

Statement and Formula

For an object fully or partially submerged in a fluid at rest, the buoyant force F_b acting upward on the object equals the weight of the displaced fluid:



The whole principle in one setup: the upward push equals the weight of the water the block pushed aside.

$$F_b = \rho_{\text{fluid}} \cdot V_{\text{displaced}} \cdot g$$

where ρ_{fluid} is the fluid's density, $V_{\text{displaced}}$ is the volume of fluid pushed aside by the object, and g is gravitational acceleration ($9.81\,\text{m/s}^2$).

Why It Works

Pressure in a fluid increases with depth. The bottom of a submerged object sits in a higher-pressure region than the top, so the upward push on the bottom exceeds the downward push on the top. The net upward force is the buoyant force. A short calculation shows this difference equals exactly $\rho g V$, the weight of the displaced fluid. The result depends only on the fluid and the volume, not on the object's shape or material.

Floating, Sinking, and Neutral Buoyancy

- **Sinks** if the object's average density exceeds the fluid's density: its weight is greater than the maximum buoyant force.
- **Floats** if its average density is less than the fluid's: it settles at a depth such that the weight of displaced fluid equals its own weight.
- **Neutrally buoyant** if the densities are equal: it stays at any depth without rising or sinking. Submarines and SCUBA divers actively adjust their average density to achieve this state.

A steel ship floats not because steel is light, it's not, but because the ship's hull encloses a large volume of air. The *average* density of (steel hull + enclosed air) is well below water's density.

Worked Examples

Example 1: Wooden block in water. A wooden cube of side 0.10 m and density 600 kg/m³ floats in water ($\rho = 1000\ \text{kg/m}^3$). At equilibrium, weight = buoyant force, so $600 \cdot 0.001 \cdot g = 1000 \cdot V_{\text{sub}} \cdot g$. Solving: $V_{\text{sub}} = 6 \times 10^{-4}\ \text{m}^3$, which is 60% of the cube's volume. The cube floats with 60% submerged and 40% above water, matching the density ratio.

Example 2: Iceberg. Ice density is $917\ \text{kg/m}^3$, seawater density $1025\ \text{kg/m}^3$. The fraction submerged is $917/1025 \approx 0.895$. Roughly 90% of an iceberg sits below the waterline, the source of the iceberg metaphor.

Example 3: Apparent weight underwater. A 5.0 kg rock of density $2500\ \text{kg/m}^3$ has volume $5/2500 = 2 \times 10^{-3}\ \text{m}^3$. Submerged in water, the buoyant force is $1000 \cdot 2 \times 10^{-3} \cdot 9.81 = 19.6$ N. Its real weight is $5 \cdot 9.81 = 49$ N. Apparent weight in water: $49 - 19.6 \approx 29$ N, feels 40% lighter.

Applications

- **Ship design.** Naval architects calculate displacement tonnage from the volume of water a hull pushes aside.
- **Hydrometers.** Floating instruments that measure liquid density (urine specific gravity, battery acid,

alcohol content in spirits) work entirely by Archimedes' principle.

- **Hot-air balloons.** Hot air inside the envelope is less dense than the surrounding cool air; the difference produces lift equal to the weight of displaced cool air minus the weight of the heated air and the balloon.
- **Submarines.** Ballast tanks are flooded with seawater to add weight (sink) or blown clear with compressed air to displace water (rise).
- **Density measurement.** Archimedes legendarily used the principle to verify the gold content of King Hiero's crown, comparing weight in air with apparent weight in water gave the density.

Related study notes: Pascal's Law, Density, Newton's Laws of Motion, Pressure.

Practice Questions

Work each question before reading its solution. The set runs from direct recall and substitution to the applied questions that exams actually use to separate grades.

Q1. State Archimedes' principle and give the buoyant-force formula.

SOLUTION

A body in a fluid feels an upward force equal to the weight of the fluid it displaces: $F_b = \rho_{fluid}\, g\, V_{displaced}$. Only the displaced volume and the FLUID's density matter; the object's own material enters the story only when comparing F_b with its weight.

Q2. A 2,000 cm^3 block is fully submerged in water. Find the buoyant force.

SOLUTION

$V = 2 \times 10^{-3}\ \mathrm{m}^3$, water density $1000\ \mathrm{kg/m^3}$: $F_b = 1000 \times 9.8 \times 0.002 = 19.6\ \mathrm{N}$, the weight of 2 kg of water. Whatever the block is made of, iron or foam, the buoyant force is the same while fully submerged.

Q3. That same block weighs 50 N in air. What does a spring scale read with the block submerged?

SOLUTION

Apparent weight $= 50 - 19.6 = 30.4\ \mathrm{N}$. Submerged weighing measures true weight minus buoyancy, which is exactly how Archimedes could test the king's crown: measure weight in air and in water, and the loss reveals the volume, hence the density.

Q4. Give the flotation condition in terms of densities, and predict: ice ($917\ \mathrm{kg/m^3}$) in water, and in ethanol ($789\ \mathrm{kg/m^3}$).

SOLUTION

A solid floats when its average density is below the fluid's. Ice floats in water (917 < 1000) with about 92% of its volume submerged, but sinks in ethanol (917 > 789). Same ice, different verdicts: buoyancy is a comparison, not a property of the object alone.

Q5. What fraction of a floating iceberg ($917\ \mathrm{kg/m^3}$) hides below seawater ($1025\ \mathrm{kg/m^3}$)?

SOLUTION

Floating equilibrium: $\rho_{ice} V_{total} = \rho_{sea} V_{sub}$, so $V_{sub}/V_{total} = 917/1025 \approx 0.89$. About 89% below, 11% visible: the literal tip of the iceberg. The submerged fraction is always the density ratio.

Q6. A steel ship floats though steel is 8 times denser than water. Resolve the paradox.

SOLUTION

The hull encloses mostly air, so the ship's AVERAGE density, steel plus enclosed air over the hull volume, drops below water's. The hull displaces a huge water volume whose weight matches the ship's. Load the ship and it settles deeper, displacing more; overload it past the point where required displacement exceeds hull volume, and it founders. Shape buys displacement; displacement buys lift.

Q7. A 60-tonne barge floats in a lock. How much water does it displace?

SOLUTION

Floating means F_b equals weight, so displaced water weighs exactly 60 tonnes: about 60 m^3 of fresh water. A floating body always displaces its own WEIGHT in fluid; a sunken body displaces only its own VOLUME. That distinction is the whole subject in one sentence.

Q8. A crown weighs 25.0 N in air and 23.6 N in water. Is it pure gold (19,300 kg/m^3)?

SOLUTION

Buoyant force $=$ 1.4 N, so displaced volume $= 1.4/(1000 \times 9.8) = 1.43 \times 10^{-4}$ m^3. Mass $= 25/9.8 = 2.55$ kg, density $= 2.55/1.43 \times 10^{-4} \approx 17{,}800$ kg/m^3. Below gold's 19,300: the crown is alloyed. This is the legendary bath-time measurement, done with a spring scale.

Q9. Why does a helium balloon rise, and why does it stop rising?

SOLUTION

Air is a fluid too: the balloon displaces air weighing more than the balloon-plus-helium, so the net buoyant force points up. With altitude the air thins, the displaced air weighs less, and at the altitude where buoyancy equals total weight the balloon hovers: neutral buoyancy. Submarines run the same equation in water, trading ballast for altitude.

Q10. A boat carrying a rock floats in a small pool. The rock is thrown overboard and sinks. Does the pool level rise, fall, or stay the same?

SOLUTION

It falls. Aboard, the rock displaces its WEIGHT in water, a large volume for a dense object. Sunk, it displaces only its VOLUME. The displacement shrinks by the difference, so the water level drops. The classic trick question, and the float-versus-sink displacement rule answers it cleanly.

Answer Key

QUICK ANSWERS

Q1 $F_b = \rho g V$ of displaced fluid **Q2** 19.6 N **Q3** 30.4 N **Q4** floats if $\rho_{obj} < \rho_{fluid}$; floats in water, sinks in ethanol **Q5** $\approx$ 89%

Q6 average density with enclosed air < water **Q7** 60 tonnes $\approx$ 60 m^3 **Q8** $\rho \approx$ 17,800: not pure gold **Q9** air buoyancy exceeds weight until density balance **Q10** falls: weight-displacement becomes volume-displacement

Revision Sheet

The principle

$F_b = \rho_{fluid}\, g\, V_{displaced}$: upthrust = weight of displaced fluid.

Apparent weight

Scale reading = true weight − F_b. The loss measures volume, hence density.

Float or sink

Compare densities. Floats if $\rho_{avg} < \rho_{fluid}$; submerged fraction = ρ_{obj}/ρ_{fluid}.

Ships and balloons

Enclosed air lowers average density; helium balloons float on air the same way.

Two displacement rules

Floating: displaces its weight. Sunk: displaces its volume. Never confuse them.

Iceberg number

917/1025 $\approx$ 89% of an iceberg is under seawater.

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