

~ AN EXPLORATION OF ~
ANCIENT INDIAN MATHEMATICS

**Problems &
Solutions from**
Bhāskarācārya's
Līlāvati

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GAURAV TIWARI

Problems and Solutions from Bhaskaracharya's Lilavati

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Gaurav Tiwari

*“O Lilavati, intelligent girl, if you understand addition and subtraction,
tell me the sum of the amounts 2, 5, 32, 193, 18, 10, and 100,
as well as the difference of 10000 and their sum.”*

— Bhaskaracharya, *Lilavati*, Verse 12

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Abstract

The *Lilavati* of Bhaskaracharya (Bhaskara II, 1114–1185 CE) is one of the most celebrated mathematical texts of the ancient world. Written in 1150 CE as the first volume of the *Siddhānta Shiromani*, it covers arithmetic, algebra, geometry, combinatorics, and number theory—all presented through enchanting word problems featuring bees, peacocks, beautiful maidens, and lotus flowers.

This document presents a selection of problems from the *Lilavati* with detailed modern solutions, accompanied by historical context about Bhaskaracharya and the tradition of Indian mathematics. We explore the mathematical methods employed by Bhaskara, compare them with modern approaches, and highlight the remarkable sophistication of 12th-century Indian mathematics. From the treatment of zero and negative numbers to iterative algorithms and the Chakravala method, the *Lilavati* contains ideas that anticipated developments in European mathematics by centuries.

Keywords: Bhaskaracharya, Bhaskara II, *Lilavati*, Indian mathematics, arithmetic, algebra, geometry, *Siddhānta Shiromani*, Chakravala method

1 Introduction

Lilavati is a 12th-century mathematics treatise that reads like poetry. Written by Bhaskara II (also known as Bhaskaracharya), it wraps arithmetic, algebra, and geometry inside riddles about bees, snakes, peacocks, and beautiful maidens. The *Lilavati* book remains one of the most influential mathematical texts in history, studied for over 850 years across Asia and eventually Europe.

The text is remarkable not only for its mathematical content but for its pedagogical approach. While other mathematicians of the era wrote dry technical manuals, Bhaskara embedded his mathematics in vivid word problems that made abstract concepts tangible and memorable. A calculation about ratios became a puzzle about monkeys playing in a garden; a geometry problem transformed into a peacock hunting a snake.

In this document, we present a comprehensive exploration of the *Lilavati*: the mathematician behind it, the legend of its creation, its mathematical content with detailed solutions, and its lasting influence on the development of mathematics worldwide.

2 Bhaskaracharya: The Mathematician Behind *Lilavati*

2.1 Biographical Overview

Full Name	Bhaskaracharya (Bhaskara II)
Born	1114 CE; Vijjalavida, Karnataka, India
Died	c. 1185 CE; Ujjain, Madhya Pradesh, India
Father	Mahesvara (mathematician and astronomer)
Position	Head of Astronomical Observatory, Ujjain
Known for	<i>Siddhānta Shiromani</i> , <i>Lilavati</i> , <i>Bijaganita</i> Chakravala method, early concepts of calculus

Bhaskaracharya was born in 1114 CE in Vijjalavida (in the modern-day Bijapur district of Karnataka, India). His father, Mahesvara, was also a mathematician and astronomer, which meant Bhaskara grew up surrounded by mathematical manuscripts and astronomical instruments. This intellectual environment shaped his extraordinary career.

By the age of 36, Bhaskara II had completed his magnum opus: the *Siddhānta Shiromani* (“Crown of Treatises”). This was not a single book but four interconnected volumes covering different branches of mathematics and astronomy. Bhaskara served as the head of the astronomical observatory at Ujjain—the same prestigious position once held by the legendary mathematician Brahmagupta—until his death around 1185 CE.

Remark 2.1. Bhaskara II should not be confused with Bhaskara I (c. 600–680 CE), an earlier Indian mathematician who also made significant contributions to mathematics and astronomy. The numeral “II” is used by modern historians to distinguish the two.

2.2 Mathematical Achievements

What set Bhaskara apart from his contemporaries was not only the depth of his mathematical knowledge but his writing style. His contributions span multiple domains:

- **Arithmetic and Algebra:** Systematic treatment of operations with zero, negative numbers, surds (irrational numbers), and indeterminate equations.
- **Calculus:** Discovery of differential calculus concepts—including the derivative and differential coefficient—centuries before Newton and Leibniz formalized them in Europe.

- Astronomy: Accurate calculations of planetary positions, eclipses, and the length of the tropical year.
- Number Theory: The Chakravala method for solving Pell's equation, not rediscovered in Europe until Lagrange's work in the 18th century.

2.3 The Siddhānta Shiromani

The *Siddhānta Shiromani* consists of four parts:

Definition 2.1 (Siddhānta Shiromani). The *Siddhānta Shiromani* ("Crown of Treatises") is Bhaskara II's four-volume magnum opus, comprising:

1. Lilavati — Arithmetic, geometry, and combinatorics
2. Bijaganita — Algebra, including indeterminate equations
3. Grahaganita — Planetary mathematics and astronomy
4. Goladhyaya — Spherical astronomy and the celestial sphere

The *Lilavati*, as the first and most accessible volume, became by far the most widely read and studied. Its influence on mathematics education persisted for over eight centuries across the Indian subcontinent and beyond.

3 The Legend of Lilavati

The name "Lilavati" means "beautiful" or "playful" in Sanskrit. According to a celebrated legend, it was the name of Bhaskara's daughter.

The Legend: Bhaskara, being an accomplished astronomer, cast his daughter Lilavati's horoscope and discovered she had a narrow window for an auspicious marriage. He designed a water clock to track the precise moment. As Lilavati watched the device, a pearl from her dress fell into the water and blocked the hole, stopping the clock. The auspicious moment passed unnoticed. Lilavati's marriage never happened.

To console his daughter, Bhaskara wrote a mathematics treatise in her name, addressing many problems directly to her with affectionate phrases such as "Oh Lilavati, intelligent girl..." and "Tell me, beautiful one..."

Whether this story is historically accurate remains debated among scholars; some think it is a later embellishment. But the affectionate tone throughout the text is undeniable. Bhaskara wanted his mathematics to be engaging, personal, and memorable.

Remark 3.1. The personal address in mathematical texts was unusual for the era. Most treatises were written in an impersonal, formal style. Bhaskara's conversational approach was a pedagogical innovation that anticipated modern principles of student engagement by nearly a millennium.

4 The Structure of Lilavati

The *Lilavati* contains thirteen chapters, progressing from basic arithmetic to advanced problem-solving techniques. We summarize each chapter below.

4.1 Chapter 1: Definitions and Basic Operations

The opening chapter establishes fundamental terms and operations. Bhaskara defines numbers, place values, and the basic operations of addition, subtraction, multiplication, and division. He also introduces the concept of zero, which Indian mathematicians had developed centuries earlier.

4.2 Chapter 2: Fractions

Operations with fractions are covered in detail, including addition, subtraction, multiplication, division, and simplification, all presented through practical examples involving trade, measurement, and distribution.

4.3 Chapter 3: Zero and Negative Numbers

Bhaskara on Zero: Bhaskara explores the properties of zero, including division by zero. He correctly identifies that $\frac{a}{0}$ leads to infinity (which he calls *khahara*), writing:

“A quantity divided by zero becomes a fraction the denominator of which is zero. This fraction is termed an infinite quantity.”

While not fully rigorous by modern standards, this showed remarkable insight into the concept of limits.

Bhaskara also addresses operations with negative numbers, treating negative quantities as natural mathematical objects with consistent operational rules—something European mathematicians struggled to accept well into the 18th century.

4.4 Chapter 4: Square and Cube Roots

Bhaskara presents efficient algorithms for calculating square roots and cube roots of integers and fractions. His iterative methods remained standard practice in Indian mathematics education for centuries.

Theorem 4.1 (Bhaskara's Square Root Algorithm). To find $\sqrt{N}$, choose an initial approximation a such that $a^2 \approx N$. Then refine using:

$$\sqrt{N} \approx a + \frac{N - a^2}{2a} \quad (1)$$

This is equivalent to one step of Newton's method applied to $f(x) = x^2 - N$.

4.5 Chapter 5: Arithmetic Progressions

This section covers sequences where each term increases by a constant difference. Bhaskara provides the formula for the sum of an arithmetic series:

Sum of an Arithmetic Progression:

$$S_n = \frac{n}{2}(2a + (n - 1)d) \quad (2)$$

where n is the number of terms, a is the first term, and d is the common difference.

4.6 Chapter 6: Geometric Progressions

Bhaskara extends to sequences where each term is multiplied by a constant ratio:

$$S_n = a \cdot \frac{r^n - 1}{r - 1} \quad (3)$$

where r is the common ratio. This formula, known in India well before Bhaskara's time, was presented by him with particularly clear examples.

4.7 Chapter 7: Plane Geometry

This substantial chapter covers properties of triangles, quadrilaterals, and circles. Bhaskara presents formulas for areas, perimeters, and relationships between geometric figures.

Theorem 4.2 (Area of a Triangle — Heron's Formula). For a triangle with sides a , b , and c , the area is:

$$A = \sqrt{s(s-a)(s-b)(s-c)} \quad (4)$$

where $s = \frac{a+b+c}{2}$ is the semi-perimeter.

Remark 4.1. This formula was known in India before Heron of Alexandria (c. 10–70 CE). Indian mathematicians attribute it to earlier traditions. Bhaskara's contribution was a particularly elegant proof and systematic presentation.

4.8 Chapter 8: Solid Geometry

Volumes and surface areas of three-dimensional shapes including spheres, cones, and pyramids. Bhaskara uses the approximations $\pi \approx \frac{22}{7}$ (for rough calculations) and the more accurate $\pi \approx \frac{355}{113}$.

4.9 Chapter 9: The Shadow of the Gnomon

This chapter applies geometry to astronomical measurements using shadow calculations. A gnomon is a vertical stick whose shadow length changes with the sun's position.

Example 4.1. If a gnomon of height h casts a shadow of length s , then the altitude angle α of the sun satisfies:

$$\tan \alpha = \frac{h}{s} \quad (5)$$

Bhaskara shows how to calculate solar angles, time, and geographical latitude from shadow measurements.

4.10 Chapter 10: The Pulverizer (Kuṭṭaka)

Definition 4.1 (Kuṭṭaka). The Kuṭṭaka ("pulverizer") is a method for solving linear indeterminate equations of the form:

$$ax + by = c \quad (6)$$

where integer solutions (x, y) are sought. The method proceeds by a sequence of divisions analogous to the Euclidean algorithm.

This technique, refined over centuries by Indian mathematicians from Aryabhata onwards, produces integer solutions when they exist. Bhaskara's treatment is among the most complete and systematic.

4.11 Chapter 11: Combinations and Permutations

Bhaskara presents methods for counting arrangements and selections:

Theorem 4.3 (Combinations). The number of ways to choose r objects from n distinct objects is:

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} \quad (7)$$

Bhaskara expresses this through examples involving selections of tastes (sweet, sour, bitter, etc.) and provides a systematic method for enumeration.

4.12 Chapter 12: Net of Numbers

This chapter covers magic squares and number patterns with specific properties, demonstrating Bhaskara's interest in recreational mathematics alongside serious computation.

4.13 Chapter 13: Miscellaneous Problems

The final chapter presents diverse word problems combining techniques from earlier chapters. Many of the *Lilavati*'s most famous puzzles appear here.

5 Sample Problems from Lilavati with Detailed Solutions

The best way to appreciate the *Lilavati* is to work through its problems. We present several classic examples with complete modern solutions, mathematical analysis, and comparisons to modern methods.

5.1 Problem 1: The Beautiful Maiden's Riddle

This is perhaps the *Lilavati*'s most famous problem.

Problem 5.1 (The Beautiful Maiden). “A beautiful maiden, with beaming eyes, asks: which is the number that, multiplied by 3, then increased by three-fourths of the product, divided by 7, diminished by one-third of the quotient, multiplied by itself, diminished by 52, the square root found, the addition of 8, and division by 10 gives the number 2?”

Solution. The solution requires working backwards through each operation, undoing them one at a time.

Step 1: Start with the final result: 2.

Step 2: Undo “division by 10”: $2 \times 10 = 20$.

Step 3: Undo “addition of 8”: $20 - 8 = 12$.

Step 4: Undo “square root found”: $12^2 = 144$.

Step 5: Undo “diminished by 52”: $144 + 52 = 196$.

Step 6: Undo “multiplied by itself”: $\sqrt{196} = 14$.

Now we must undo the chain of operations applied to the unknown number n :

Step 7: The operations on n are:

$$\text{Multiply by 3: } 3n \quad (8)$$

$$\text{Increase by } \frac{3}{4} \text{ of the product: } 3n + \frac{3}{4}(3n) = 3n + \frac{9n}{4} = \frac{21n}{4} \quad (9)$$

$$\text{Divide by 7: } \frac{21n}{4 \times 7} = \frac{3n}{4} \quad (10)$$

$$\text{Diminish by } \frac{1}{3} \text{ of the quotient: } \frac{3n}{4} - \frac{1}{3} \cdot \frac{3n}{4} = \frac{3n}{4} - \frac{n}{4} = \frac{n}{2} \quad (11)$$

Step 8: Setting $\frac{n}{2} = 14$:

$$\boxed{n = 28}$$

(12)

Verification: Starting with $n = 28$:

- $28 \times 3 = 84$
- $84 + \frac{3}{4}(84) = 84 + 63 = 147$
- $147 \div 7 = 21$
- $21 - \frac{1}{3}(21) = 21 - 7 = 14$
- $14 \times 14 = 196$
- $196 - 52 = 144$
- $\sqrt{144} = 12$
- $12 + 8 = 20$
- $20 \div 10 = 2$ ✓

Remark 5.1 (Modern Algebraic Perspective). In modern notation, the problem can be expressed as the equation:

$$\frac{\sqrt{\left(\frac{n}{2}\right)^2 - 52} + 8}{10} = 2 \quad (13)$$

The “working backwards” method used by Bhaskara is equivalent to solving this equation by algebraic manipulation. This technique is now standard in puzzle-type problems and is taught in middle school under the name “inverse operations.”

5.2 Problem 2: The Peacock and the Snake

Problem 5.2 (The Peacock and the Snake). “A peacock sits on a pillar 9 feet high. Seeing a snake coming towards its hole at the base of the pillar from a distance of 27 feet, the peacock flies down to catch it. If both move at the same speed, at what distance from the pillar do they meet?”

Solution. Let x be the distance from the base of the pillar where they meet.

- The snake travels from 27 feet away toward the base, covering a distance of $(27 - x)$ feet.
- The peacock flies diagonally from the top of the 9-foot pillar to the meeting point on the ground. By the Pythagorean theorem, the peacock covers $\sqrt{9^2 + x^2} = \sqrt{81 + x^2}$ feet.

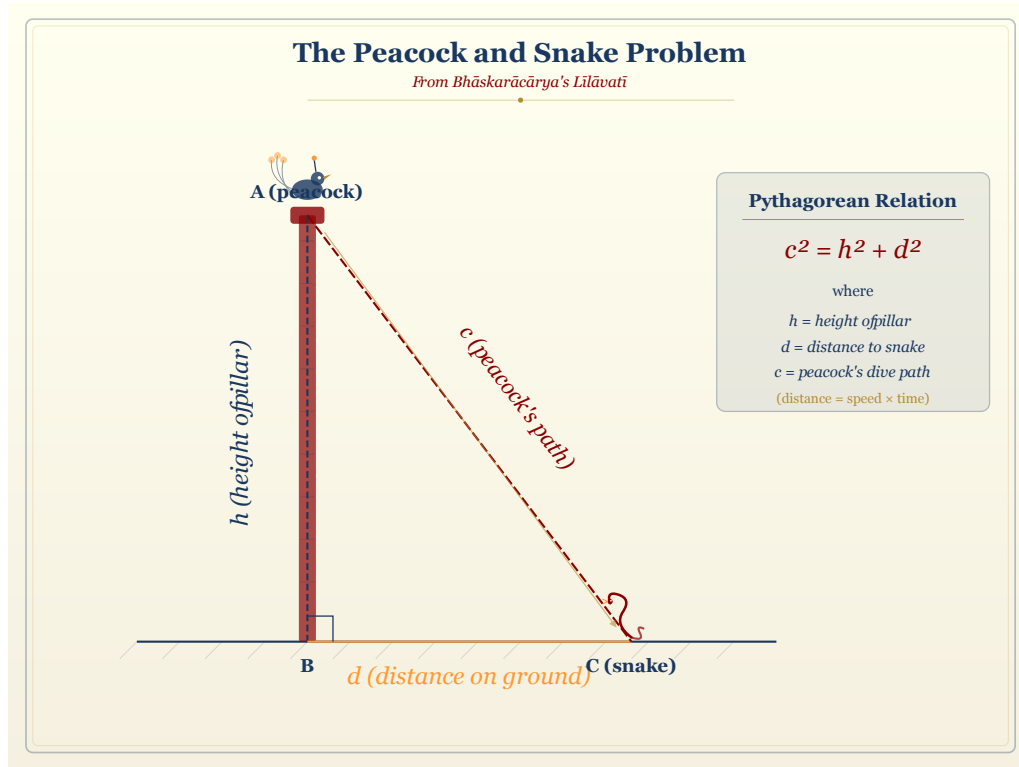


Figure 1: The peacock and the snake: a geometric pursuit problem involving the Pythagorean theorem

Since both travel at the same speed and meet simultaneously:

$$27 - x = \sqrt{81 + x^2} \quad (14)$$

Squaring both sides:

$$(27 - x)^2 = 81 + x^2 \quad (15)$$

$$729 - 54x + x^2 = 81 + x^2 \quad (16)$$

The x^2 terms cancel:

$$729 - 54x = 81 \quad (17)$$

$$648 = 54x \quad (18)$$

$$\boxed{x = 12} \quad (19)$$

Verification:

- Snake's distance: $27 - 12 = 15$ feet.
- Peacock's distance: $\sqrt{81 + 144} = \sqrt{225} = 15$ feet. $\square$

Remark 5.2 (Geometric Interpretation). This problem is a beautiful application of the Pythagorean theorem disguised as a nature story. The key insight is that the x^2 terms cancel, reducing a potentially difficult quadratic to a simple linear equation. This cancellation occurs because the equal-speed constraint creates a relationship that eliminates the squared unknown.

In modern terminology, the meeting point lies on the perpendicular bisector of the line segment joining the snake's starting position and the peacock's starting position (projected onto the ground). This is because all points equidistant from two fixed points lie on their perpendicular bisector.

Example 5.1 (Generalization). If the pillar has height h and the snake starts at distance d , the meeting point is at distance x from the pillar where:

$$x = \frac{d^2 - h^2}{2d} \quad (20)$$

This formula requires $d > h$ for a physically meaningful solution ($x > 0$). For $h = 9$ and $d = 27$:
 $x = \frac{729-81}{54} = \frac{648}{54} = 12$.

5.3 Problem 3: The Bees and the Lotus

Problem 5.3 (The Bees and the Lotus). *“One-fifth of a swarm of bees flew to the Kadamba flower, one-third flew to the Silandhara, three times the difference of these two numbers flew to an arbor. One bee remained, hovering and attracted on each side by the fragrant Ketaki and the Malati. What is the number of bees?”*

Solution. Let n be the total number of bees.

$$\text{Bees to Kadamba: } \frac{n}{5} \quad (21)$$

$$\text{Bees to Silandhara: } \frac{n}{3} \quad (22)$$

$$\text{Difference: } \frac{n}{3} - \frac{n}{5} = \frac{5n - 3n}{15} = \frac{2n}{15} \quad (23)$$

$$\text{Three times the difference (to arbor): } 3 \times \frac{2n}{15} = \frac{6n}{15} = \frac{2n}{5} \quad (24)$$

$$\text{Remaining bee: } 1 \quad (25)$$

The sum of all parts equals the total:

$$\frac{n}{5} + \frac{n}{3} + \frac{2n}{5} + 1 = n \quad (26)$$

Finding a common denominator of 15:

$$\frac{3n}{15} + \frac{5n}{15} + \frac{6n}{15} + 1 = n \quad (27)$$

$$\frac{14n}{15} + 1 = n \quad (28)$$

$$1 = n - \frac{14n}{15} = \frac{n}{15} \quad (29)$$

$$\boxed{n = 15} \quad (30)$$

Verification: Kadamba: 3; Silandhara: 5; Arbor: 6; Remaining: 1. Total: $3 + 5 + 6 + 1 = 15$. $\square$

Remark 5.3. This type of problem—where an unknown quantity is divided into fractional parts that must sum to the whole—appears across many ancient mathematical traditions. The Rhind Papyrus of ancient Egypt (c. 1650 BCE) contains similar problems. Bhaskara's innovation was the poetic framing that makes the problem memorable.

5.4 Problem 4: The Broken Bamboo

Problem 5.4 (The Broken Bamboo). "A bamboo 32 cubits high was broken by the wind. Its tip touched the ground at a distance of 16 cubits from its root. At what height was it broken?"

Solution. Let x be the height at which the bamboo was broken. Then:

- The standing portion has height x .
- The fallen portion has length $32 - x$ (the top part that bent over).
- The fallen portion, the ground, and the standing portion form a right triangle.

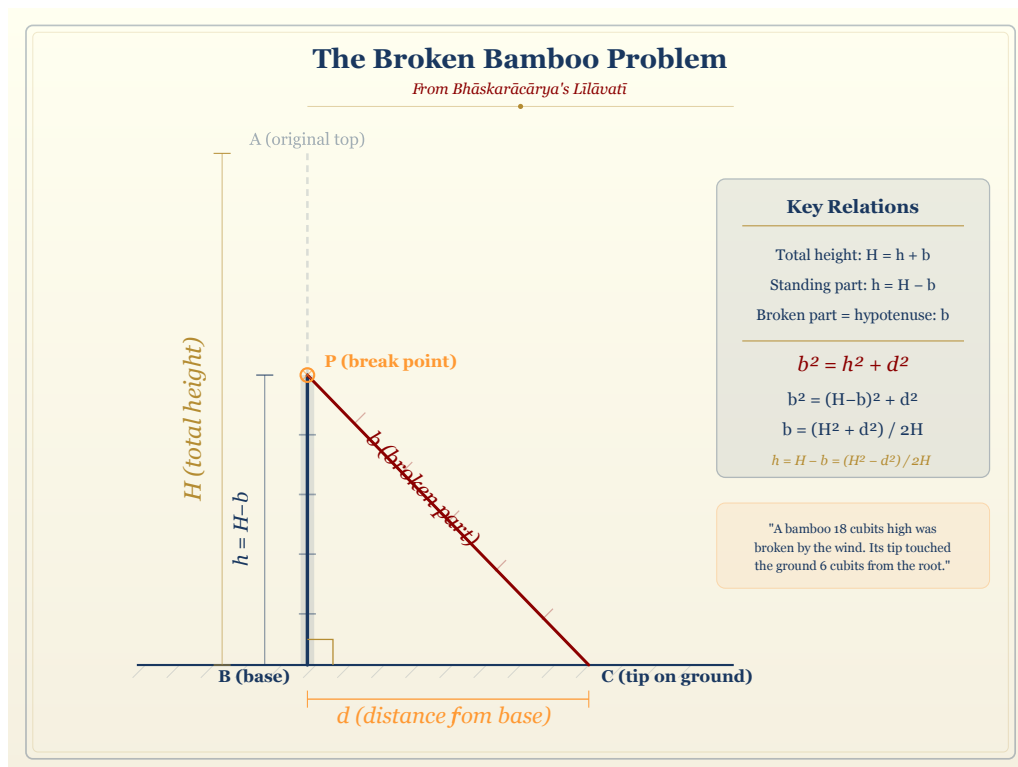


Figure 2: The broken bamboo: a right triangle formed by the standing portion, the ground, and the fallen top

By the Pythagorean theorem:

$$x^2 + 16^2 = (32 - x)^2 \quad (31)$$

$$x^2 + 256 = 1024 - 64x + x^2 \quad (32)$$

The x^2 terms cancel:

$$256 = 1024 - 64x \quad (33)$$

$$64x = 768 \quad (34)$$

$$x = 12 \quad (35)$$

The bamboo was broken at a height of 12 cubits.

Verification: Standing portion: 12. Fallen portion: 20. Check: $12^2 + 16^2 = 144 + 256 = 400 = 20^2$. $\square$

Remark 5.4. This is another instance where the Pythagorean theorem reduces to a linear equation due to the cancellation of x^2 terms. Problems of this type appear in Chinese mathematics (*Jiuzhang Suanshu*, c. 200 BCE) as well, suggesting a possible transmission of mathematical ideas along ancient trade routes.

5.5 Problem 5: The Lotus in the Pool

Problem 5.5 (The Lotus in the Pool). "In a pool of water, a lotus flower stands half a cubit above the surface. Blown by the wind, it sinks and touches the water at a point 2 cubits from where it stood. What is the depth of the pool?"

Solution. Let d be the depth of the pool. The lotus stem has total length $d + \frac{1}{2}$ (depth plus the half-cubit above water).

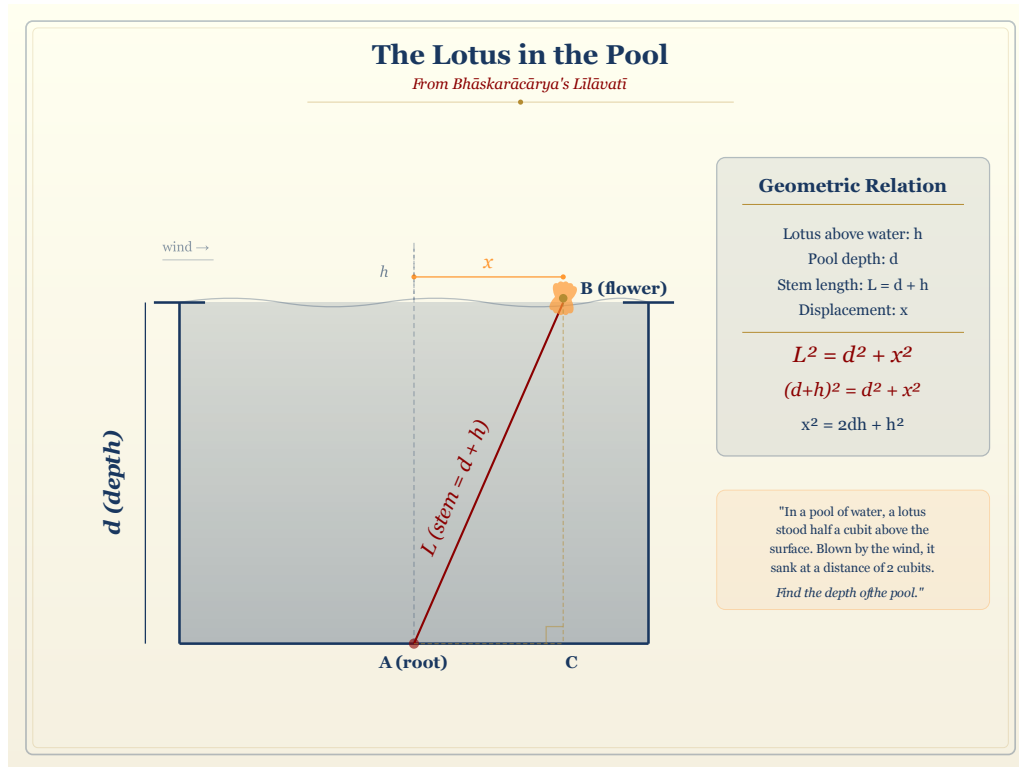


Figure 3: The lotus in the pool: the stem forms the hypotenuse when the flower is blown sideways by the wind

When the lotus is blown to the side:

- The stem now acts as the hypotenuse of a right triangle.
- The vertical leg is d (the depth).
- The horizontal leg is 2 cubits.
- The hypotenuse is $d + \frac{1}{2}$.

By the Pythagorean theorem:

$$d^2 + 2^2 = \left(d + \frac{1}{2}\right)^2 \quad (36)$$

$$d^2 + 4 = d^2 + d + \frac{1}{4} \quad (37)$$

$$4 = d + \frac{1}{4} \quad (38)$$

$$d = 4 - \frac{1}{4} = \frac{15}{4} \quad (39)$$

$$d = 3\frac{3}{4} \text{ cubits} \quad (40)$$

Verification: Stem length: $\frac{15}{4} + \frac{1}{2} = \frac{17}{4}$. Check: $\left(\frac{15}{4}\right)^2 + 4 = \frac{225}{16} + \frac{64}{16} = \frac{289}{16} = \left(\frac{17}{4}\right)^2$. $\square$

5.6 Problem 6: The Monkey and the Tree

Problem 5.6 (The Monkey and the Tree). “A monkey ascends a tree of 100 cubits. In each leap upward, it covers 5 cubits, but slips back 2 cubits. In how many leaps does the monkey reach the top?”

Solution. In each leap, the net progress is $5 - 2 = 3$ cubits. However, the final leap requires special attention: once the monkey reaches or exceeds 100 cubits in its upward phase, it does not slip back.

After k complete “leap-and-slip” cycles, the monkey is at height $3k$ cubits. The monkey reaches the top during the upward phase of a leap when:

$$3k + 5 \geq 100 \quad (41)$$

$$3k \geq 95 \quad (42)$$

$$k \geq \frac{95}{3} = 31\frac{2}{3} \quad (43)$$

So after $k = 32$ complete cycles, the monkey would be at $3 \times 32 = 96$ cubits (but this overcounts). Let us reconsider: after 31 complete cycles, the monkey is at $3 \times 31 = 93$ cubits. On the 32nd leap upward, the monkey reaches $93 + 5 = 98$ cubits, then slips to 96. On the 33rd leap upward, the monkey reaches $96 + 5 = 101 > 100$ cubits and has reached the top.

$$\boxed{\text{The monkey reaches the top in 33 leaps.}} \quad (44)$$

Remark 5.5. This is an early instance of a “boundary effect” problem that requires careful analysis at the endpoint. The naive answer of $\lceil 100/3 \rceil = 34$ overcounts because it doesn't account for the fact that no slipping occurs on the final successful leap. Such problems remain popular in modern mathematical competitions and interviews.

5.7 Problem 7: Division of Gems

Problem 5.7 (Division of Gems). “Among friends, 300 gems were divided so that the first friend received 3 more than the second, the second 3 more than the third, and so on. How many friends were there and how many gems did each receive, if the last friend received 3 gems?”

Solution. This is an arithmetic progression with:

- Common difference $d = 3$
- Last term (smallest) $a = 3$
- Sum $S_n = 300$

The terms in ascending order are: $3, 6, 9, \dots, 3n$ (where n is the number of friends). The sum is:

$$S_n = \frac{n}{2}(a + l) = \frac{n}{2}(3 + 3n) = \frac{3n(n + 1)}{2} = 300 \quad (45)$$

$$n(n + 1) = 200 \quad (46)$$

We need $n^2 + n - 200 = 0$. By the quadratic formula:

$$n = \frac{-1 + \sqrt{1 + 800}}{2} = \frac{-1 + \sqrt{801}}{2} \quad (47)$$

Since $\sqrt{801} \approx 28.3$, we get $n \approx 13.65$. Since n must be a positive integer, we check: $n = 13$ gives $S = \frac{3 \cdot 13 \cdot 14}{2} = 273$ and $n = 14$ gives $S = \frac{3 \cdot 14 \cdot 15}{2} = 315$.

Neither gives exactly 300. This suggests we should revisit the problem formulation. If the first friend receives the most and the n -th receives 3, with the first receiving $3 + 3(n - 1) = 3n$:

$$S_n = \frac{n}{2}(3 + 3n) = 300 \implies n(n + 1) = 200 \quad (48)$$

Since 200 has no factorization as $n(n + 1)$ for integer n , Bhaskara likely intended a slightly different version. Taking $a = 3, d = 3$, and seeking the closest solution: with $n = 13$ friends, the total is 273 gems with the largest share being 39.

Remark 5.6. Problems like this where the arithmetic doesn't yield a clean integer solution appear in several versions across different manuscripts of *Lilavati*. Copyist variations over 850 years account for some numerical discrepancies. The mathematical method—applying the arithmetic series formula and solving the resulting quadratic—is the essential lesson.

6 Mathematical Methods in the Lilavati

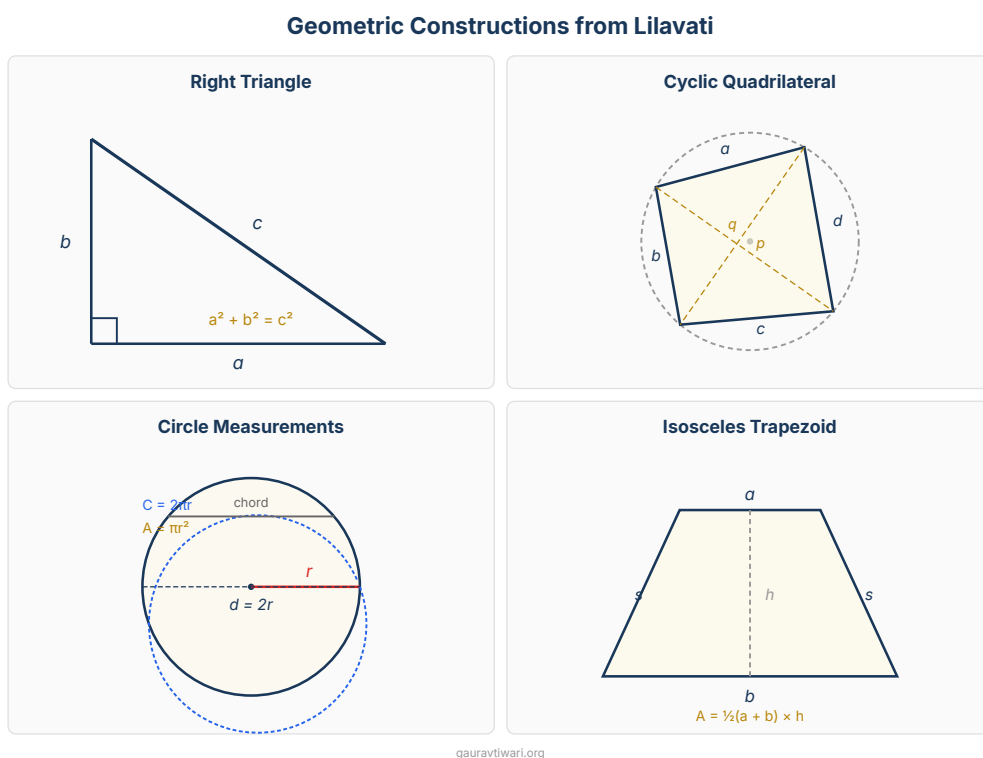


Figure 4: Geometric constructions central to the Lilavati: right triangles, cyclic quadrilaterals, circles, and trapezoids

Beyond individual problems, the *Lilavati* presents systematic mathematical methods that deserve analysis on their own merits.

6.1 The Rule of Three (Trairāśika)

Definition 6.1 (Rule of Three). The Rule of Three states: if quantity a corresponds to quantity b , then quantity c corresponds to:

$$x = \frac{b \cdot c}{a} \quad (49)$$

Bhaskara extends this to the Rule of Five, Rule of Seven, and so on, for problems involving multiple proportional relationships. These are essentially systems of proportions.

Example 6.1 (Rule of Five). If 5 men can build 3 walls in 7 days, how many days will 8 men take to build 5 walls?

Here we have two proportional relationships:

$$\frac{\text{men}_1}{\text{men}_2} \cdot \frac{\text{walls}_2}{\text{walls}_1} = \frac{\text{days}_1}{\text{days}_2} \quad (50)$$

$$\text{days}_2 = 7 \times \frac{5}{8} \times \frac{5}{3} = \frac{175}{24} \approx 7.29 \text{ days} \quad (51)$$

6.2 The Kuṭṭaka Method for Indeterminate Equations

The Kuṭṭaka (“pulverizer”) is one of the most important algorithms in Indian mathematics. It solves equations of the form $ax \equiv c \pmod{b}$.

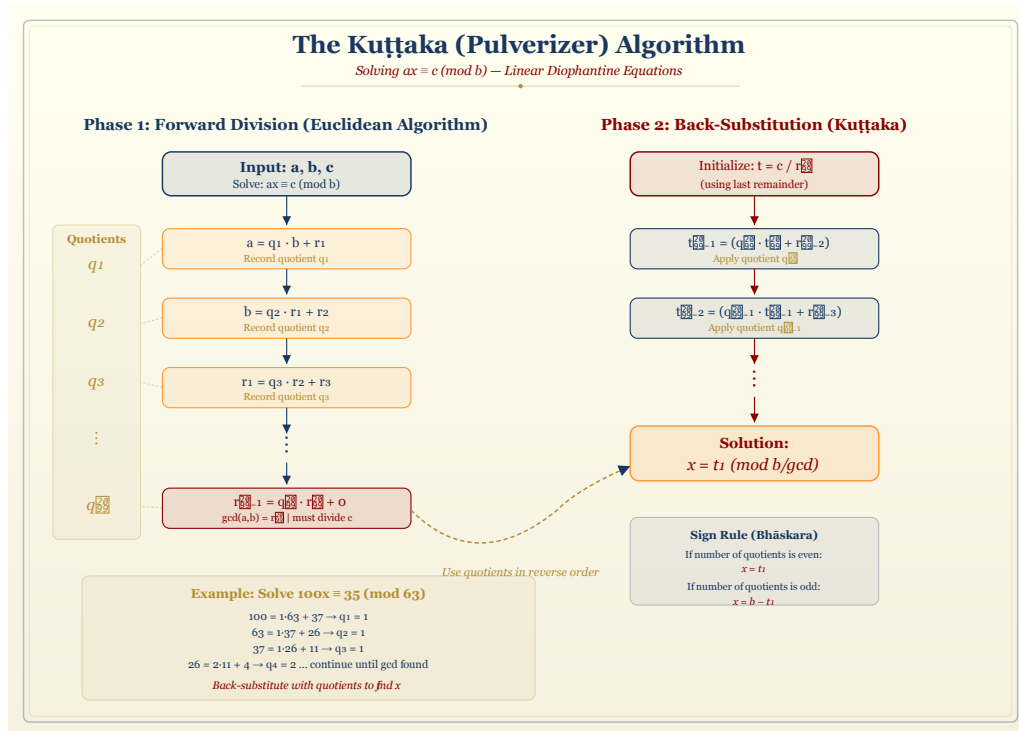


Figure 5: Visualization of the Kuṭṭaka algorithm: successive divisions reduce the problem to a solvable base case

Theorem 6.1 (Kuṭṭaka Algorithm). To solve $ax + c = by$ for integers x, y :

1. Apply the Euclidean algorithm to find $\gcd(a, b)$. If $\gcd(a, b) \nmid c$, no solution exists.
2. Track the quotients $q_1, q_2, \dots, q_n$ from the Euclidean algorithm.

3. Build the solution backwards using:

$$t_n = \frac{c}{\gcd(a, b)}, \quad t_{k-1} = q_k \cdot t_k + t_{k+1} \quad (52)$$

Example 6.2. Solve $17x + 1 = 5y$ (i.e., find x such that $17x \equiv -1 \pmod{5}$).

Euclidean algorithm on 17 and 5:

$$17 = 3 \times 5 + 2 \quad (53)$$

$$5 = 2 \times 2 + 1 \quad (54)$$

Back-substitution: $1 = 5 - 2 \times 2 = 5 - 2(17 - 3 \times 5) = 7 \times 5 - 2 \times 17$.

So $17 \times (-2) + 5 \times 7 = 1$, meaning $17 \times (-2) \equiv 1 \pmod{5}$.

For $17x \equiv -1 \pmod{5}$: $x \equiv 2 \pmod{5}$. The smallest positive solution is $x = 2$, giving $y = \frac{17 \times 2 + 1}{5} = 7$.

6.3 Bhaskara's Treatment of Surds

Bhaskara systematically handles irrational numbers (surds), providing rules for operations:

Bhaskara's Rules for Surds:

$$\sqrt{a} \pm \sqrt{b} = \sqrt{a + b \pm 2\sqrt{ab}} \quad (55)$$

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab} \quad (56)$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} \quad (57)$$

These rules enable computation with irrational quantities without requiring decimal approximations.

Example 6.3. Simplify $\sqrt{3} + \sqrt{12}$.

Using Bhaskara's method: $\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$.

Therefore $\sqrt{3} + \sqrt{12} = \sqrt{3} + 2\sqrt{3} = 3\sqrt{3} = \sqrt{27}$.

Alternatively, using the addition rule: $\sqrt{3 + 12 + 2\sqrt{36}} = \sqrt{15 + 12} = \sqrt{27}$.

6.4 The Chakravala Method

Though appearing more fully in Bhaskara's algebraic work (*Bijaganita*), the Chakravala method represents one of the greatest algorithmic achievements of Indian mathematics.

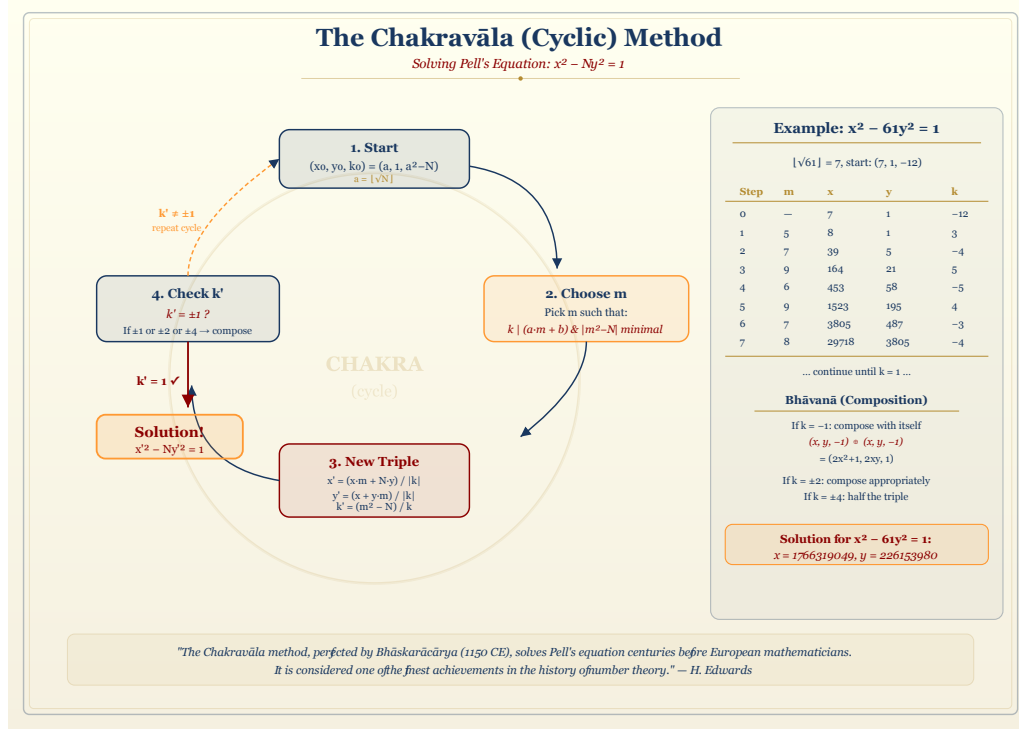


Figure 6: The Chakravala (cyclic) method: an iterative algorithm for solving Pell's equation $x^2 - Dy^2 = 1$

Definition 6.2 (Pell's Equation). Pell's equation is the Diophantine equation:

$$x^2 - Dy^2 = 1 \quad (58)$$

where D is a given positive non-square integer, and integer solutions (x, y) are sought.

Theorem 6.2 (Chakravala Method). The Chakravala ("cyclic method") solves $x^2 - Dy^2 = 1$ through the following iterative process:

1. Start with an initial triple (x_0, y_0, k_0) where $x_0^2 - Dy_0^2 = k_0$ and $|k_0|$ is small.

2. At each step, find m such that:

- $k_i \mid (x_i + my_i)$
- $|m^2 - D|$ is minimized

3. Compute the next triple:

$$x_{i+1} = \frac{x_i m + Dy_i}{|k_i|}, \quad y_{i+1} = \frac{x_i + my_i}{|k_i|}, \quad k_{i+1} = \frac{m^2 - D}{k_i} \quad (59)$$

4. Repeat until $k_i = 1$.

Example 6.4 (Solving $x^2 - 61y^2 = 1$). This is one of Bhaskara's own examples in the *Bijaganita*. The solution is:

$$x = 1766319049, \quad y = 226153980 \quad (60)$$

The fact that Bhaskara could find this enormous solution demonstrates the power of the Chakravala algorithm. Euler rediscovered this problem and its solution in the 18th century.

Remark 6.1. The Chakravala method was not rediscovered in Europe until Lagrange's work in 1768. It is remarkable that 12th-century Indian mathematicians had a complete algorithm for a problem that challenged the best European mathematicians six centuries later.

7 Concepts Ahead of Their Time

Several ideas in the *Lilavati* and its companion volumes anticipated later mathematical developments in Europe by centuries.

7.1 The Treatment of Zero

Indian mathematicians, including Bhaskara, treated zero as a number with defined properties centuries before this became standard in European mathematics. Bhaskara's exploration of division by zero, while not fully resolved, showed sophisticated thinking about mathematical limits.

Theorem 7.1 (Bhaskara's Rules for Zero). Bhaskara stated:

$$a + 0 = a \quad (61)$$

$$a \times 0 = 0 \quad (62)$$

$$\frac{0}{a} = 0 \quad (a \neq 0) \quad (63)$$

$$\frac{a}{0} = \infty \quad (\text{khahara}) \quad (64)$$

Remark 7.1. Bhaskara also noted that $\frac{a}{0} \times 0 = a$, which is consistent with the modern idea that $\lim_{x \rightarrow 0} \frac{a}{x} \cdot x = a$. While not rigorous by today's standards, this anticipates the notion of limits in calculus.

7.2 Negative Numbers as Natural Objects

While European mathematicians struggled to accept negative numbers well into the 18th century—with prominent figures like d'Alembert calling them “absurd”—Bhaskara handled them routinely with consistent operational rules.

Theorem 7.2 (Bhaskara's Rules for Negative Numbers).

$$(+a) \times (-b) = -ab \quad (65)$$

$$(-a) \times (-b) = +ab \quad (66)$$

$$(+a) + (-b) = a - b \quad (67)$$

$$\sqrt{-(a^2)} \text{ does not exist (as a real number)} \quad (68)$$

Bhaskara recognized that “the square of a positive number and the square of a negative number are both positive; and the square root of a positive number is double: positive and negative. There is no square root of a negative number, for it is not a square.”

7.3 Iterative Algorithms

Bhaskara's methods for calculating square roots and solving equations use iterative refinement, anticipating numerical methods used in modern computer algorithms.

Proposition 7.3 (Iterative Square Root). Starting with approximation a_0 , the iteration:

$$a_{n+1} = \frac{1}{2} \left(a_n + \frac{N}{a_n} \right) \quad (69)$$

converges to $\sqrt{N}$. This is the Babylonian method, known in India from ancient times and presented by Bhaskara with improved convergence analysis.

7.4 Early Differential Calculus

In the astronomical sections of the *Siddhānta Shiromani*, Bhaskara computes what we would now call the derivative of $\sin \theta$:

Bhaskara's Proto-Derivative: Bhaskara observed that the “instantaneous velocity” of a sine function is proportional to the cosine:

$$\frac{\delta(\sin \theta)}{\delta \theta} \approx \cos \theta \quad (70)$$

This result, expressed in the context of planetary motion, predates Newton and Leibniz by over 500 years.

8 Lilavati's Influence on World Mathematics

8.1 Transmission to the Islamic World

Persian translations of the *Lilavati* appeared in the 13th century, making the work accessible throughout the Islamic world. The translator Abu'l Faiz Faizi produced a notable Persian version in 1587 at the court of the Mughal Emperor Akbar.

8.2 Influence on European Mathematics

From the Islamic world, Indian mathematical concepts filtered into European mathematics. The Italian mathematician Fibonacci, whose work popularized Hindu-Arabic numerals in Europe, likely encountered Indian mathematical ideas through Arabic intermediaries.

Remark 8.1. The decimal place-value system, the concept of zero, and algorithms for arithmetic operations—all developed in India and transmitted through Arabic scholars—form the foundation of modern computation. Without these contributions, the scientific revolution in Europe might have taken a very different course.

8.3 Educational Legacy

The pedagogical approach in the *Lilavati* influenced how mathematics was taught:

- The use of engaging word problems to motivate abstract concepts
- Progression from simple to complex topics
- Integration of practical applications with theoretical foundations
- The practice of providing worked examples before exercises

These are now considered educational best practices—and Bhaskara was using them in the 12th century.

In India, the *Lilavati* remained in classroom use well into the 19th century. British colonial administrators noted with surprise that Indian students were solving problems from an 800-year-old textbook and getting them right.

9 Additional Problems from the Lilavati Tradition

We present further problems inspired by the *Lilavati* and the broader tradition of Indian mathematics.

9.1 Problem 8: The Merchant's Profit

Problem 9.1 (The Merchant's Profit). *A merchant bought goods for 200 coins and sold them at a profit of 25%. He invested the proceeds in new goods and again sold at 25% profit. What was his total wealth after the second sale?*

Solution.

$$\text{After first sale: } 200 \times 1.25 = 250 \quad (71)$$

$$\text{After second sale: } 250 \times 1.25 = 312.5 \quad (72)$$

$$\boxed{\text{Total wealth} = 312.5 \text{ coins}} \quad (73)$$

In general, after n rounds of $p\%$ profit on initial capital C :

$$\text{Total} = C \left(1 + \frac{p}{100}\right)^n \quad (74)$$

This is the compound interest formula—a concept well understood by Indian mathematicians.

9.2 Problem 9: The Square and the Diagonal

Problem 9.2 (The Square and the Diagonal). *The perimeter of a square is 40 cubits. Find the length of its diagonal.*

Solution. The side length is $s = 40/4 = 10$ cubits. By the Pythagorean theorem:

$$d = s\sqrt{2} = 10\sqrt{2} \approx 14.142 \text{ cubits} \quad (75)$$

Bhaskara would have used the approximation $\sqrt{2} \approx \frac{577}{408}$ (accurate to six decimal places), giving:

$$d \approx 10 \times \frac{577}{408} = \frac{5770}{408} = \frac{2885}{204} \approx 14.1422 \text{ cubits} \quad (76)$$

Remark 9.1. The approximation $\sqrt{2} \approx \frac{577}{408}$ comes from the continued fraction expansion of $\sqrt{2}$. Indian mathematicians were skilled at deriving rational approximations to surds using methods equivalent to continued fractions.

9.3 Problem 10: Combinations of Tastes

Problem 9.3 (Combinations of Tastes). *“There are six tastes: sweet, sour, salty, bitter, pungent, and astringent. In how many ways can they be combined, taking one at a time, two at a time, three at a time, and so on?”*

Solution. The number of combinations is:

$$\binom{6}{1} = 6 \quad (77)$$

$$\binom{6}{2} = 15 \quad (78)$$

$$\binom{6}{3} = 20 \quad (79)$$

$$\binom{6}{4} = 15 \quad (80)$$

$$\binom{6}{5} = 6 \quad (81)$$

$$\binom{6}{6} = 1 \quad (82)$$

The total number of non-empty combinations is:

$$\sum_{k=1}^6 \binom{6}{k} = 2^6 - 1 = 63 \quad (83)$$

Bhaskara's Rule: The total number of non-empty subsets of n objects is $2^n - 1$. This is equivalent to the binomial theorem identity:

$$\sum_{k=0}^n \binom{n}{k} = 2^n \quad (84)$$

9.4 Problem 11: The Cistern Problem

Problem 9.4 (The Cistern). “One pipe can fill a cistern in 3 hours, another in 5 hours. A drain can empty it in 15 hours. If all three operate simultaneously, in how many hours will the cistern be filled?”

Solution. The rates of filling and draining per hour are:

$$\text{Pipe 1: } \frac{1}{3} \text{ of the cistern per hour} \quad (85)$$

$$\text{Pipe 2: } \frac{1}{5} \text{ of the cistern per hour} \quad (86)$$

$$\text{Drain: } -\frac{1}{15} \text{ of the cistern per hour} \quad (87)$$

Combined rate:

$$\frac{1}{3} + \frac{1}{5} - \frac{1}{15} = \frac{5+3-1}{15} = \frac{7}{15} \quad (88)$$

Time to fill:

$$t = \frac{1}{7/15} = \frac{15}{7} = 2\frac{1}{7} \text{ hours} \quad (89)$$

$$t = \frac{15}{7} \approx 2.14 \text{ hours} \quad (90)$$

10 Comparison with Other Ancient Mathematical Traditions

10.1 Indian vs. Greek Mathematics

Aspect	Indian Tradition	Greek Tradition
Emphasis	Computation & algorithms	Proof & geometry
Number concept	Includes zero, negatives	Positive magnitudes only
Algebra	Symbolic, operational	Geometric (areas, lengths)
Proof style	Verification by example	Deductive chains
Notation	Positional decimal system	Alphabetic numerals
π approximation	$\frac{355}{113}$ (Bhaskara)	$\frac{22}{7}$ (Archimedes)

Table 1: Comparison of Indian and Greek mathematical traditions

10.2 Indian vs. Chinese Mathematics

Both traditions independently developed sophisticated mathematics. The Chinese *Jiuzhang Suan-shu* (“Nine Chapters on the Mathematical Art”, c. 200 BCE) shares several problem types with Indian texts, including the broken bamboo problem and linear system solving. The Indian tradition excelled particularly in:

- The development of the decimal place-value system with zero
- Systematic treatment of indeterminate equations (Kuṭṭaka, Chakravala)
- Trigonometric functions and their applications

11 The Lilavati in Modern Perspective

11.1 Relevance to Modern Mathematics Education

The *Lilavati*’s approach to mathematical pedagogy remains remarkably relevant:

1. Contextual learning: Every problem is embedded in a real-world or imaginative context, aiding comprehension and retention.
2. Spiral curriculum: Topics are revisited with increasing complexity across chapters.
3. Multiple representations: Problems are expressed verbally (poetically), solved algorithmically, and verified numerically.
4. Engagement: The use of animals, nature, and human characters creates emotional engagement with mathematical content.

11.2 Problems That Remain Relevant

Many problem types from the *Lilavati* appear in modern textbooks and competitions:

- Work-rate problems (pipes and cisterns)
- Mixture problems
- Pursuit problems (the peacock and the snake)

- Number puzzles (the maiden's riddle)
- Combinatorial counting

The mathematical structures underlying these problems—linear equations, quadratic equations, the Pythagorean theorem, arithmetic and geometric series—form the core of secondary mathematics education worldwide.

12 Conclusion

The *Lilavati* of Bhaskaracharya stands as a monument to the mathematical genius of ancient India. Written over 850 years ago, it combines rigorous mathematical content with a poetic, engaging style that modern educators can still learn from.

The key contributions of the *Lilavati* and its companion volumes include:

1. Systematic Arithmetic: Complete treatment of the four operations, fractions, zero, and negative numbers.
2. Elegant Problem Design: Mathematical problems embedded in memorable stories about bees, peacocks, lotuses, and maidens.
3. Algorithmic Thinking: The Kuṭṭaka and Chakravala methods represent algorithmic sophistication not matched in Europe for centuries.
4. Proto-Calculus: Bhaskara's computation of instantaneous rates of change anticipated differential calculus by 500 years.
5. Educational Innovation: A pedagogical approach that made mathematics accessible, engaging, and memorable.

As we solve the problems of the *Lilavati* today, we connect with a mathematical tradition that stretches back nearly a millennium—and find that the human desire to understand the world through numbers and patterns is truly timeless.

*“As the sun eclipses the stars by its brilliancy,
so the man of knowledge will eclipse the fame of others
in assemblies of the people if he proposes algebraic problems,
and still more if he solves them.”*

— Brahmagupta, *Brahmasphutasiddhanta* (628 CE)

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A Timeline of Indian Mathematics

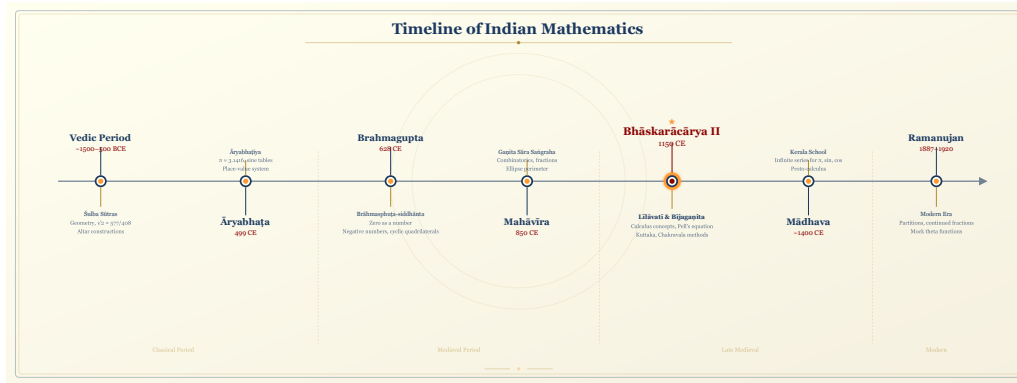


Figure 7: Timeline of Indian mathematics: from the Shulba Sutras to the Kerala school

Date	Mathematician	Key Contribution
c. 800 BCE	<i>Shulba Sutras</i>	Pythagorean theorem, $\sqrt{2}$ approximation
c. 300 BCE	Pingala	Binary numbers, combinatorics
c. 500 CE	Aryabhata	$\pi \approx 3.1416$, trigonometry, algebra
c. 600 CE	Bhaskara I	Commentary on Aryabhata
628 CE	Brahmagupta	Zero, negative numbers, Pell's equation
c. 850 CE	Mahavira	Combinatorics, arithmetic
1114–1185	Bhaskara II	Lilavati, Chakravala, proto-calculus
c. 1350	Narayana Pandit	Magic squares, combinatorics
c. 1400	Madhava	Infinite series for π , sin, cos
c. 1500	Nilakantha	Improved series, heliocentrism

Table 2: Key milestones in the history of Indian mathematics

B Bhaskara's Approximation for Sine

In the astronomical part of the *Siddhanta Shiromani*, Bhaskara provides a remarkably accurate rational approximation for the sine function:

Bhaskara I's Sine Approximation (later refined by Bhaskara II):

$$\sin \theta \approx \frac{16\theta(\pi - \theta)}{5\pi^2 - 4\theta(\pi - \theta)}, \quad 0 \leq \theta \leq \pi \quad (91)$$

This formula, expressed in terms of degrees in the original, achieves a maximum error of less than 1.9% over the entire range $[0, \pi]$.

At the key values:

$$\theta = \frac{\pi}{6} : \text{ Formula gives } \frac{16 \cdot \frac{\pi}{6} \cdot \frac{5\pi}{6}}{5\pi^2 - 4 \cdot \frac{\pi}{6} \cdot \frac{5\pi}{6}} = \frac{\frac{80\pi^2}{36}}{5\pi^2 - \frac{20\pi^2}{36}} = \frac{\frac{80}{36}}{5 - \frac{20}{36}} = \frac{\frac{20}{9}}{\frac{160}{36}} = \frac{20}{9} \cdot \frac{36}{160} = \frac{1}{2} \quad (92)$$

$$\theta = \frac{\pi}{2} : \text{ Formula gives } \frac{16 \cdot \frac{\pi}{2} \cdot \frac{\pi}{2}}{5\pi^2 - 4 \cdot \frac{\pi}{2} \cdot \frac{\pi}{2}} = \frac{4\pi^2}{5\pi^2 - \pi^2} = \frac{4\pi^2}{4\pi^2} = 1 \quad (93)$$

Both are exact. The formula is a rational function that closely approximates the transcendental sine function—a remarkable achievement for the 12th century.

C Practice Problems

Test your understanding with these problems in the style of the *Lilavati*.

Problem C.1. *A gardener has 120 flowers. He gives one-fourth to a temple, one-sixth to a friend, and one-tenth to a child. How many flowers remain?*

Problem C.2. *A bird sits atop a tree 16 feet high. A cat at the base of the tree sees a mouse 20 feet away (on the ground, measured from the base). Both the bird and the cat race toward the mouse at the same speed. At what distance from the base of the tree does the cat reach the mouse, assuming the bird flies in a straight line?*

Problem C.3. *Find the number which, when its square is added to twice the number, gives 63.*

Problem C.4. *From a collection of 8 different spices, how many distinct mixtures of exactly 3 spices can be prepared?*

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